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Assessment and improvement of structural safety under seismic actions of existing constructions:
Historic Buildings and R.C. Structures

SEMINAR

R.C. STRUCTURES:

LIMITS OF THE EXISTING TOOLS
FOR THE ANALYSIS OF STRUCTURAL RESPONSE TO STATIC AND DYNAMIC ACTIONS

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Beer Sheva, Shamoom College

International Conservation Center Città di Roma
Israel Antiquities Authority רשות העתיקות
ועדת ההיגוי הבין-משרדית להיערכות לרעידות אדמה
National Steering Committee for Earthquake Preparedness
SCE
איגוד המהנדסים

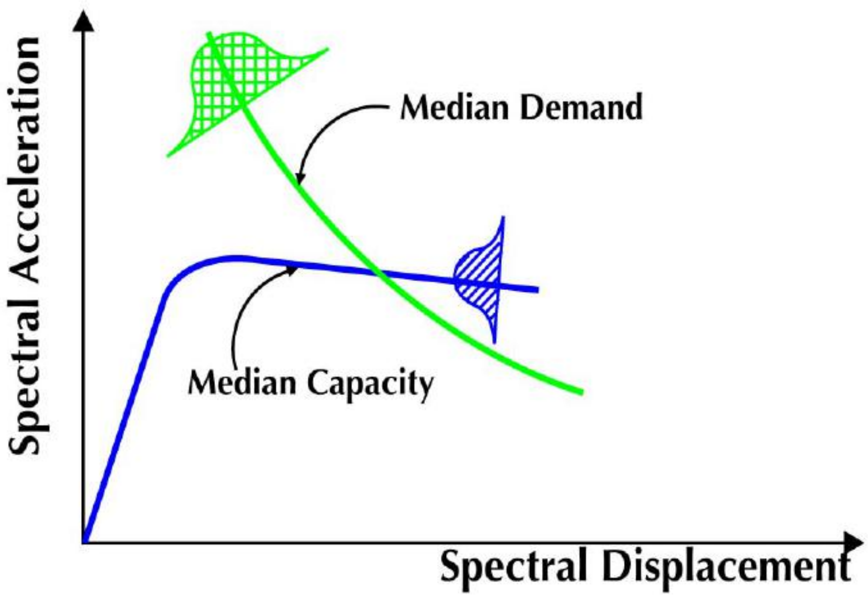
Summary

- 1. LINEAR ANALYSES
- 2. NON-LINEAR STATIC AND DYNAMIC ANALYSES
- 3. DISPLACEMENT-BASED METHODS
- 4. PROBABILISTIC APPROACHES FOR SEISMIC RISK ASSESSMENT

Uncertainty in

seismic action, site condition → **Seismic DEMAND**

structural properties → **Structural CAPACITY**



"Structural response to strong earthquake ground motions cannot be accurately predicted due to **large uncertainties and the randomness of structural properties and ground motion parameters**.
Consequently, excessive sophistication in structural analysis is not warranted."
(P. Fajfar, 2002)

The usual questions that should be addressed when deciding the type of analysis to perform are

- What is the goal of the analysis?
- What are the acceptable amounts of error?

Introduction

The analytical methods used for modelling the seismic behavior of structures, can be grouped into four categories:

LINEAR STATIC (LATERAL FORCE METHOD) ANALYSIS- LSA:

May be applied to buildings whose response is not significantly affected by contributions from modes of vibration higher than the fundamental mode in each principal direction.

RESPONSE SPECTRUM ANALYSIS –RSA:

Genneralized linear method for design and assessment. This type of analysis shall be applied to buildings which do not satisfy the conditions for applying the lateral force method of analysis.

LINEAR PROCEDURES

DISPLACEMENT BASED METHOD

(Design/Assessment methods)

NON LINEAR STATIC ANALYSIS- NSA (PUSHOVER):

Non-linear analysis carried out under conditions of constant gravity loads and monotonically increasing horizontal loads.

NON-LINEAR TIME HISTORY ANALYSIS- NTHA:

The time-dependent response of the structure may be obtained through direct numerical integration of its differential equations of motion, using the accelerograms defined in 3.2.3.1 to represent the ground motions.

NON-LINEAR PROCEDURES

(Verification/Assessment methods)

Introduction

LINEAR METHODS

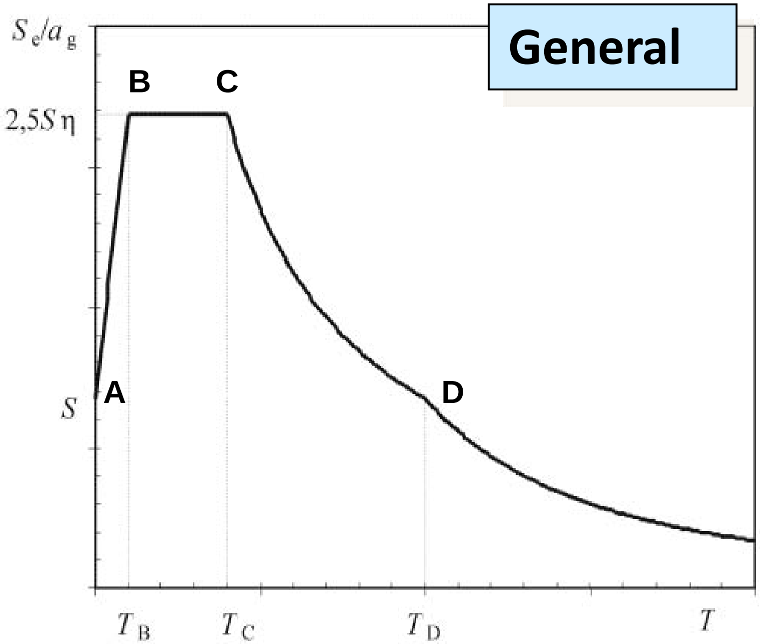
Linear procedures provide an elastic analysis and subsequent calculation of the deformations and stresses in each element. These are then corrected by appropriate coefficients, to take account of the effects of non-linearity, and compared with limit values corresponding to the item type and level of performance sought. The analysis provides results that can be unreliable if the behavior is conditioned by the strong penetration in the plastic range of some elements and the consequent redistribution of the forces due to the premature failure of these elements as occurs for example in the case of irregular structures, for the presence of concentrated ductility demands ;

NON-LINEAR METRHODS

The procedures involve static (push-over) or dynamic analyses (step by step in time-history). The former applies horizontal forces to the structure, that are increased monotonically until the building reaches the failure. The latter (NDA, THA) provides for the direct integration of the equation of motion. Both require the modeling of the inelastic structural behaviour. In these approaches, the designer can rely on resistance sources and energy dissipation that are not are not explicitely considered in procedures based on the analysis elastic. Nonlinear analyses allows a more accurate assessment of the expected response, e.g. required in the case of seismic verification of existing structures.

- 1. LINEAR ANALYSES

The seismic action depends on the seismic zone and on the nature of the supporting ground, information that is contained into the response spectrum:



From: EUROCODICE 8

Ground types	S	T_B	T_C	T_D
A	1,0	0,15	0,40	2,0
B, C, E	1,25	0,15	0,50	2,0
D	1,35	0,20	0,80	2,0

$F_h = W \cdot S_d(T)$

$S_d(T) = S_e(T)/q$ design response spectrum

q = behaviour factor

$S_e(T)$ = elastic response spectrum

$0 \le T < T_B$ $\quad S_e(T) = a_g \cdot S \cdot \left(1 + \frac{T}{T_B} \cdot (\eta \cdot 2{,}5 - 1)\right)$

$T_B \le T < T_C$ $\quad S_e(T) = a_g \cdot S \cdot \eta \cdot 2{,}5$

$T_C \le T < T_D$ $\quad S_e(T) = a_g \cdot S \cdot \eta \cdot 2{,}5 \left(\frac{T_C}{T}\right)$

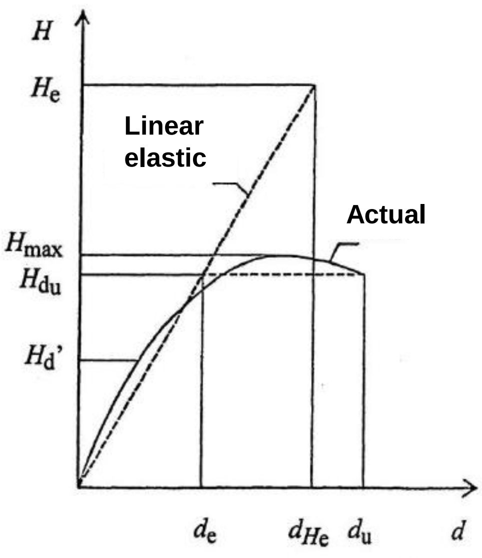
$T_D \le T$ $\quad S_e(T) = a_g \cdot S \cdot \eta \cdot 2{,}5 \cdot \left(\frac{T_C T_D}{T^2}\right)$

Design Spectrum For Elastic Analysis

The capacity of structural systems to resist seismic actions in the non-linear range generally permits their design for resistance to seismic forces smaller than those corresponding to a linear elastic response.

This capacity of the structure to dissipate energy is taken into account by performing an elastic analysis based on a response spectrum reduced by introducing the behaviour factor q.

$$\boxed{S_d(T) = S_e(T)/\,q}$$



Se(T) = elastic response spectrum

q = behaviour factor

For some categories of buildings:

STRUCTURAL TYPE	DCM	DCH
Frame system, dual system, coupled wall system	$3{,}0\,\alpha_u/\alpha_1$	$4{,}5\,\alpha_u/\alpha_1$
Uncoupled wall system	3,0	$4{,}0\,\alpha_u/\alpha_1$
Torsionally flexible system	2,0	3,0
Inverted pendulum system	1,5	2,0

Table 4.1: Consequences of structural regularity on seismic analysis and design

Regularity		Allowed Simplification		Behaviour factor
Plan	Elevation	Model	Linear-elastic Analysis	(for linear analysis)
Yes	Yes	Planar	Lateral force[a]	Reference value
Yes	No	Planar	Modal	Decreased value
No	Yes	Spatial[b]	Lateral force[a]	Reference value
No	No	Spatial	Modal	Decreased value

[a] If the condition of **4.3.3.2.1(2)**a) is also met.

[b] Under the specific conditions given in **4.3.3.1(8)** a separate planar model may be used in each horizontal direction, in accordance with **4.3.3.1(8)**.

Regularity in elevation – main conditions (EC8):

1. All lateral load resisting systems, such as cores, structural walls, or frames, shall run without interruption from their foundations to the top of the building

2. The lateral stiffness and the mass of the individual storeys shall remain constant or reduce gradually from the base to the top of the building.

When setbacks are present, there are special additional conditions (made available to limit the unfavourable effects of setbacks) that must be satisfied.

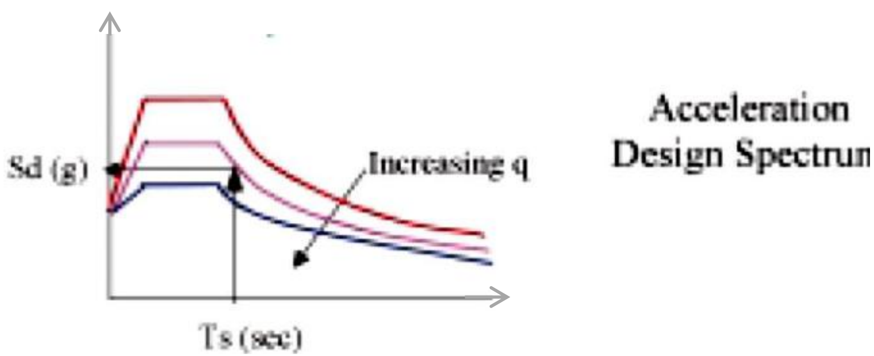
Static Method For Regular Buildings

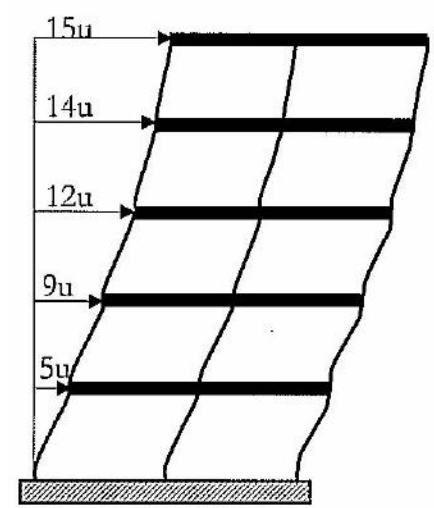
Sequence of operations required to evaluate the design base shear according to Lateral Force Analysis is the following:

1. Evaluate the fundamental period of vibration T_1
2. Select the behavior factor q
3. Get the spectral ordinate at T_1 from spectrum modified by q (design spectrum)
4. Calculate base shear as $$V_b = m \cdot \frac{S_a(T_1, \xi_{el})}{q}$$
5. Distribute the horizontal load in terms of equivalent static forces up the building in proportion to mass m_j and estimated mode shape

$$\boxed{T_1 = C_t H^{3/4}}$$

C_t = 0,075 for RC frames
H – the height of the building (in m), up to 40 m in height





Static method for regular buildings

2. Find the corresponding spectral acceleration S_a from the design response spectra

3. Calculate the base shear in the dominant mode $\boxed{F_b = S_a \sum_j m_j}$

4. Distribute the horizontal load up the building in proportion to mass m_j and estimated mode shape

$$F_j = F_b \frac{\phi_j m_j}{\sum_i \phi_i m_i} \quad \text{or} \quad \boxed{F_j = F_b \frac{z_j m_j}{\sum_i z_i m_i}}$$

when the mode shape is estimated as a straight line, z_j is the height of the j_{th} storey above the base

5. Calculate member forces and displacements d_e by static analysis

   if the forces were calculated assuming a structure ductility q, then the actual structural displacements are $d_s = q\, d_e$

Static method for regular buildings

Torsional effects

Accidental torsion, due to uncertainties in the mass and stiffness distribution, must be added to the calculated eccentricity.
Torsional effects may be accounted for by multiplying the action effects in the individual resisting elements by factor

$$\boxed{\delta = 1 + 0.6\frac{x}{L_e}}$$

x – the distance of the element from the centre of mass (perpendicular to the direction of the seismic action)

L_e – the distance between the two outermost lateral load resisting elements (perpendicular to the direction of the seismic action)

Torsional moment M^T_i at each floor equal to the story shear F_i multiplied by 5% of the floor dimension L_i, perpendicular to the direction of the seismic force.

$$\boxed{M_i^T = 0{,}05 L_i F_i}$$

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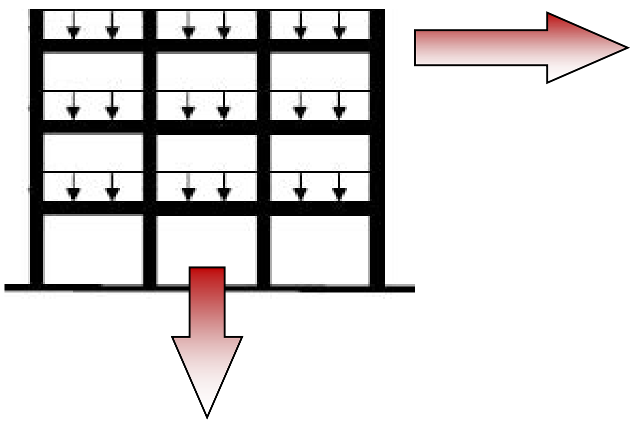
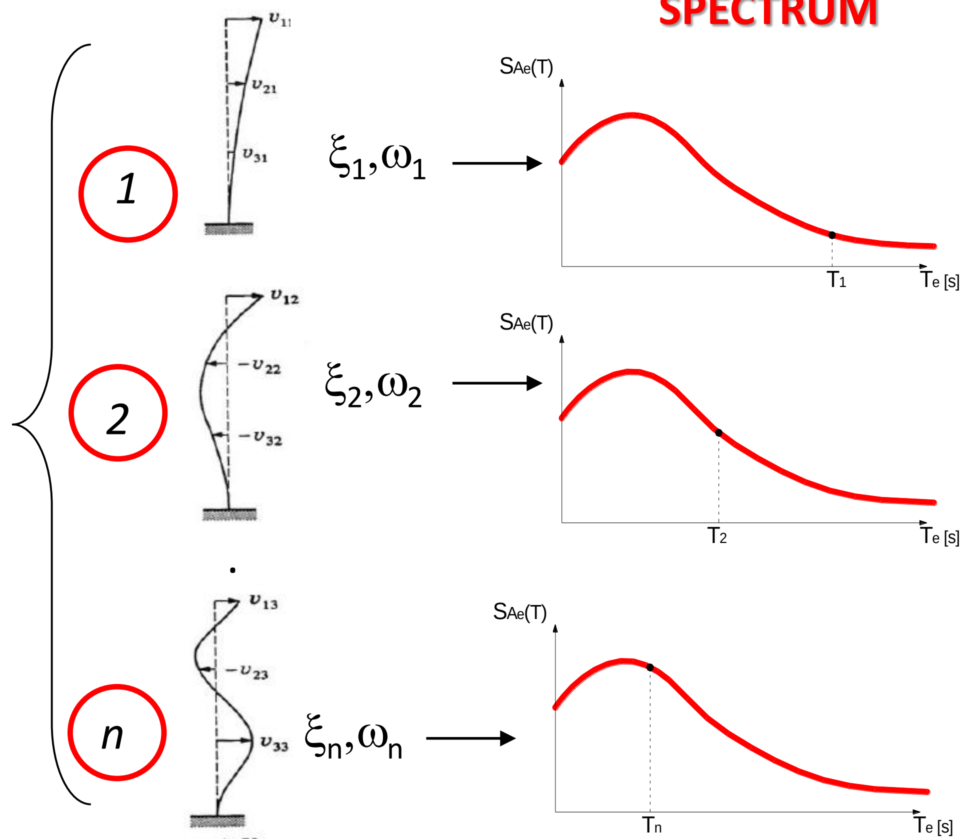
Basic principle of RSA is the possibility to uncouple the dynamic behaviour of a structure in the response of each single mode contributing to the overall response.

Equation of the dynamic equilibrium for a MDOF system:

$$M\ddot{\mathbf{x}} + C\dot{\mathbf{x}} + K\mathbf{x} = -MR\ddot{\mathbf{x}}_{\mathbf{g}}$$



MODAL ANALYSIS + **USE OF RESPONSE SPECTRUM**



To uncouple the equation s of motion,
a linear transforamtion is introduced using the modal matrix (having on the columns the eigenvectors):

$$\Phi^T M \Phi \ddot{\mathbf{u}} + \Phi^T C \Phi \dot{\mathbf{u}} + \Phi^T K \Phi \mathbf{u} = -\Phi^T M r \ddot{\mathbf{x}}_{\mathbf{g}}$$

- For each mode, a response is read from the **design spectrum**, based on the modal frequency and the modal mass, and they are then combined to provide an estimate of the total response of the structure, which is supposed to behave linearly.

➢ Input parameters:

- Elastic spectrum
- Modal analysis results
- Damping (Rayleigh: C=aM+bK)
- Combination of modal responses

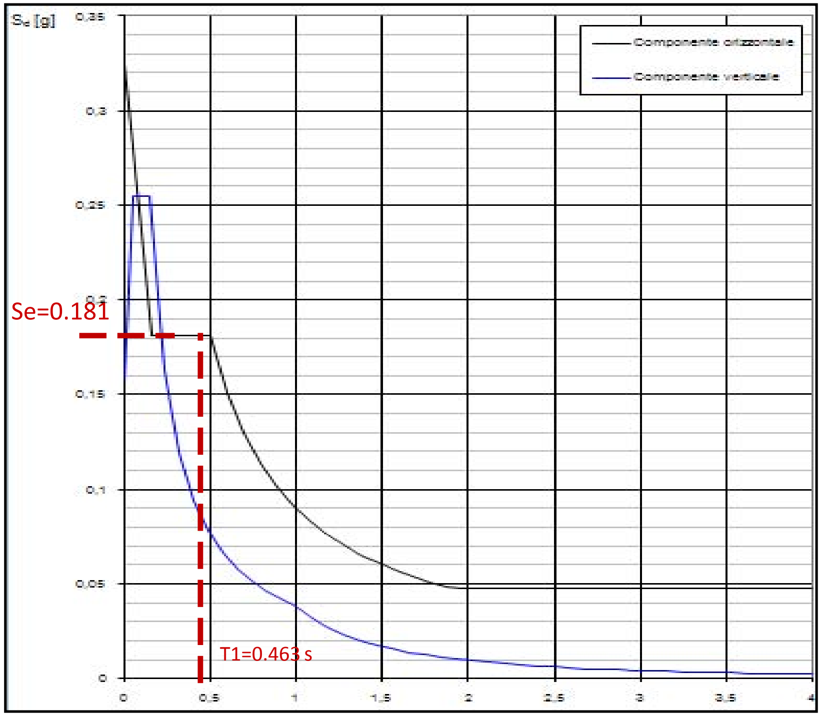
SRSS $\boxed{E=\sqrt{\sum E_r^{\,2}}}$

CQC $E=\sqrt{\sum_r \sum_s \rho_{rs} E_r E_s}$ ($T_j \leq 0.9T_i$ for $T_j<T_i$)

➢ Combination of the effects of the components of the seismic action:

$$E = E_{Ex} + 0.3E_{Ey}$$

$$E = E_{Ey} + 0.3E_{Ex}$$



Response Spectrum Analysis

- Eigenvalue problem $$|K-\omega^2 M|=0$$

- Periods and frequencies of vibration are evaluated

$$T=\frac{2\pi}{\omega}\quad [\mathrm{s}]\quad , \quad \mathrm{f}=\frac{1}{\mathrm{T}}\quad [Hz]$$

- For each *ith* mode of vibration, generalized mass, effective modal mass and modal participation factor are evaluated

$$M_i^{\,*}=\overline{\varphi}_i^{\,T}\mathbf{M}\overline{\varphi}_i \qquad \tilde{M}_i=\frac{(\varphi_i^{\,T}\mathbf{MR})^2}{M_i^{\,*}} \qquad \gamma_i=\frac{\overline{\varphi}_i^{\,T}\mathbf{M}\overline{R}}{M_i^{\,*}}$$

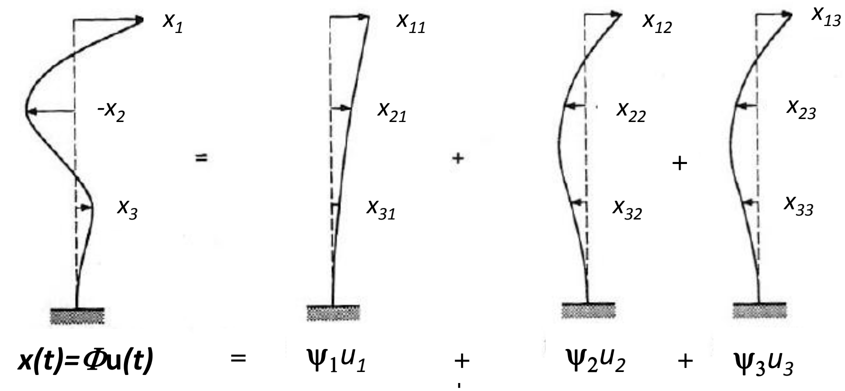
- Requirement:
  - the sum of the effective modal masses for the modes taken into account amounts to at least 85% of the total mass of the structure;
  - all modes with effective modal masses greater than 5% of the total mass are taken into account.

This transformation converts the differential equation from the real coordinates x(t)

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{0}$$

to normal coordinates u(t)

$$\mathbf{M}\boldsymbol{\Phi}\ddot{\mathbf{u}}(t)+\mathbf{K}\boldsymbol{\Phi}\mathbf{u}(t)=\mathbf{0}$$



And remembering the ortogonality conditions of the mode shapes

$$\mathbf{x}(t)=\boldsymbol{\Phi}\mathbf{u}(t)=\boldsymbol{\psi}_1 u_1+\boldsymbol{\psi}_2 u_2+\ldots+\boldsymbol{\psi}_n u_n$$

$$\boldsymbol{\Phi}^T\mathbf{M}\boldsymbol{\Phi}\ddot{\mathbf{u}}(t)+\boldsymbol{\Phi}^T\mathbf{K}\boldsymbol{\Phi}\mathbf{u}(t)=\mathbf{0}$$

$$\boldsymbol{\Phi}=\begin{vmatrix} \vdots & \vdots & & \vdots & & \vdots \\ \boldsymbol{\psi}_1 & \boldsymbol{\psi}_2 & \cdots & \boldsymbol{\psi}_r & \cdots & \boldsymbol{\psi}_N \\ \vdots & \vdots & & \vdots & & \vdots \end{vmatrix} \qquad \boldsymbol{\Omega}=\begin{vmatrix} \bar{\omega}_1^2 & 0 & \vdots & 0 \\ 0 & \bar{\omega}_2^2 & \vdots & 0 \\ \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & \vdots & \bar{\omega}_N^2 \end{vmatrix}$$

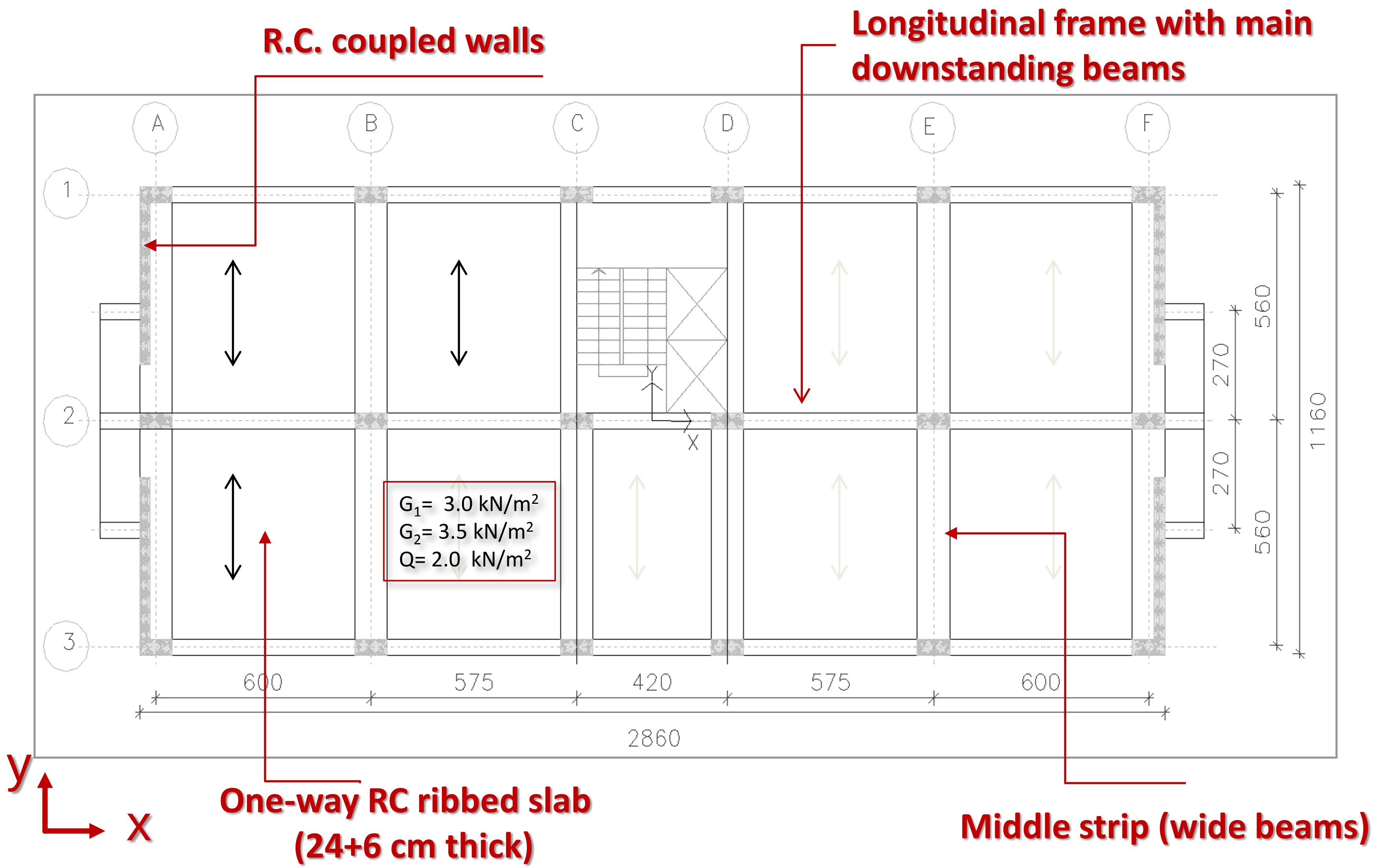
$$\tilde{\mathbf{M}}=\boldsymbol{\Phi}^T\mathbf{M}\boldsymbol{\Phi}=\mathbf{I}$$

$$\tilde{\mathbf{K}}=\boldsymbol{\Phi}^T\mathbf{K}\boldsymbol{\Phi}=\boldsymbol{\Omega}$$

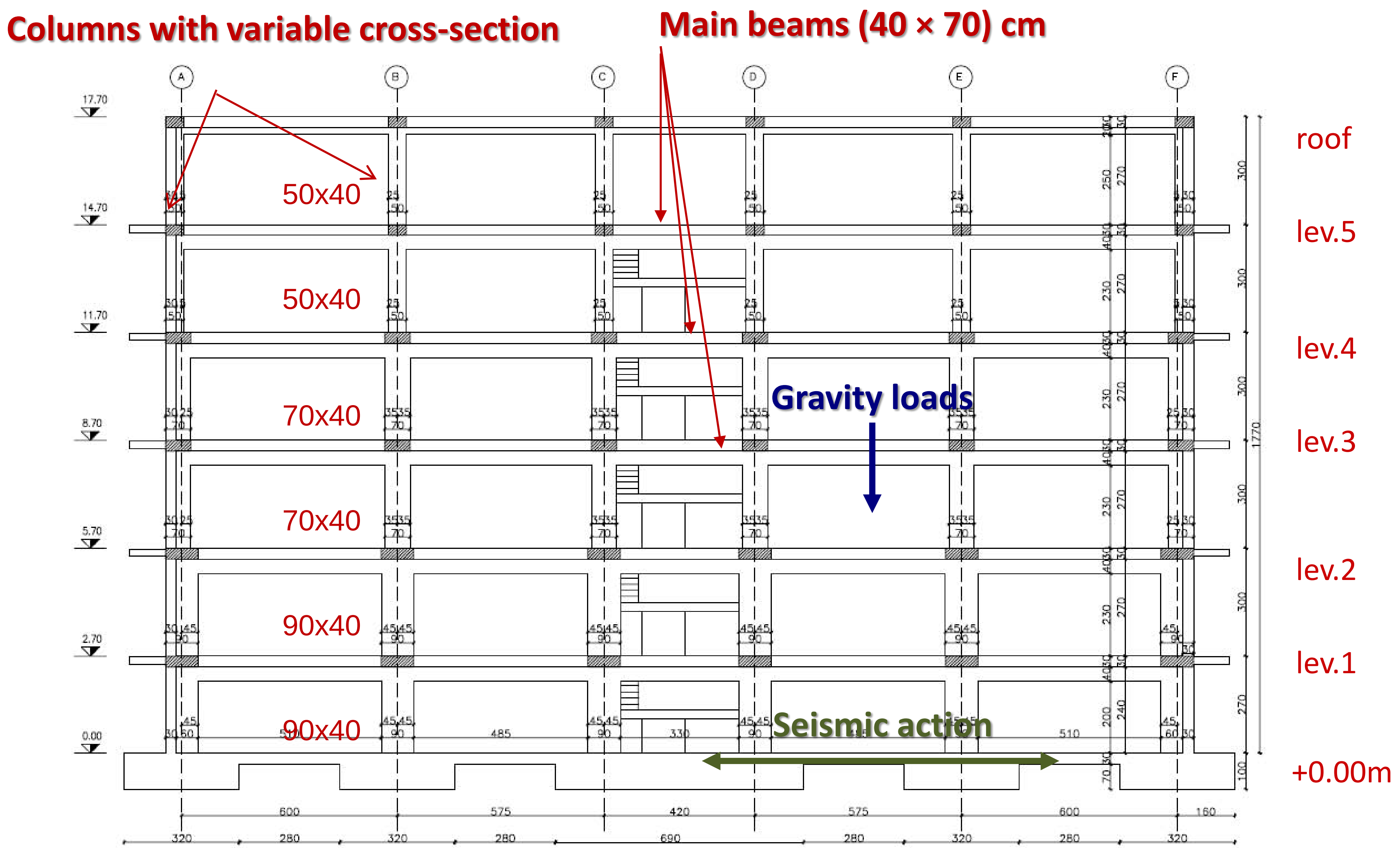
$$\tilde{\mathbf{M}}\ddot{\mathbf{u}}(t)+\tilde{\mathbf{K}}\mathbf{u}(t)=\mathbf{0} \longrightarrow \boxed{\ddot{\mathbf{u}}(t)+\boldsymbol{\Omega}\mathbf{u}(t)=\mathbf{0}} \quad \begin{cases} \ddot{u}_1+\omega_1^2 u_1=0 \\ \ldots\ldots \\ \ddot{u}_n+\omega_n^2 u_n=0 \end{cases}$$

representing a system of differential equations that are uncopled, in which the response can be found separately for each dgree of freedom (normal coordinate u_i)

Case study: multi-storey RC frame

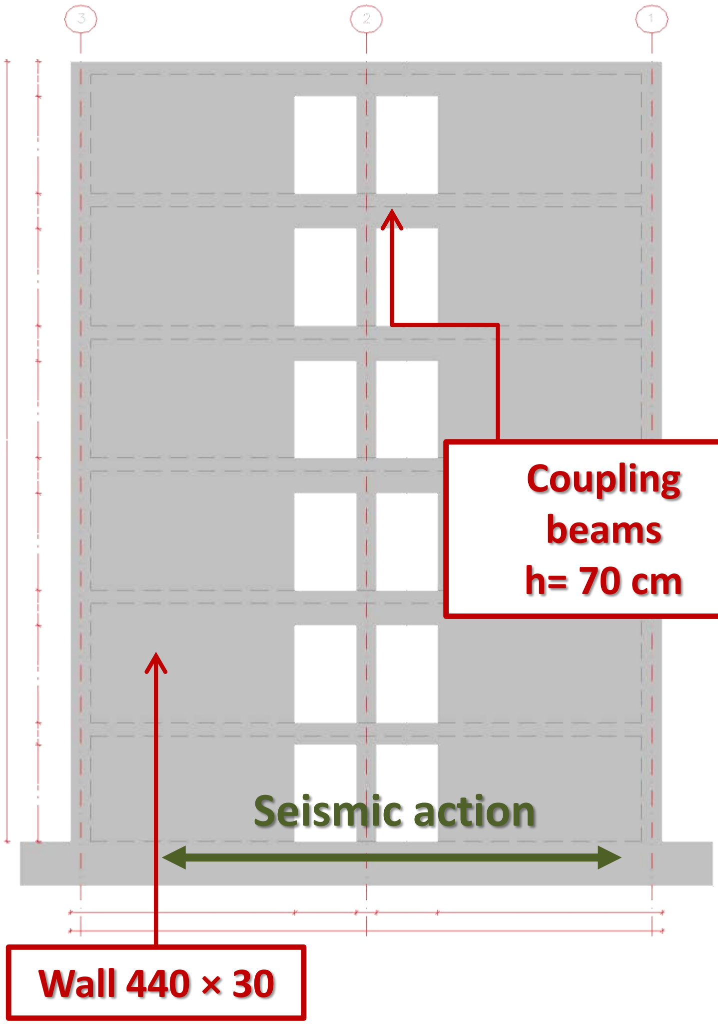


Case study: multi-storey RC frame

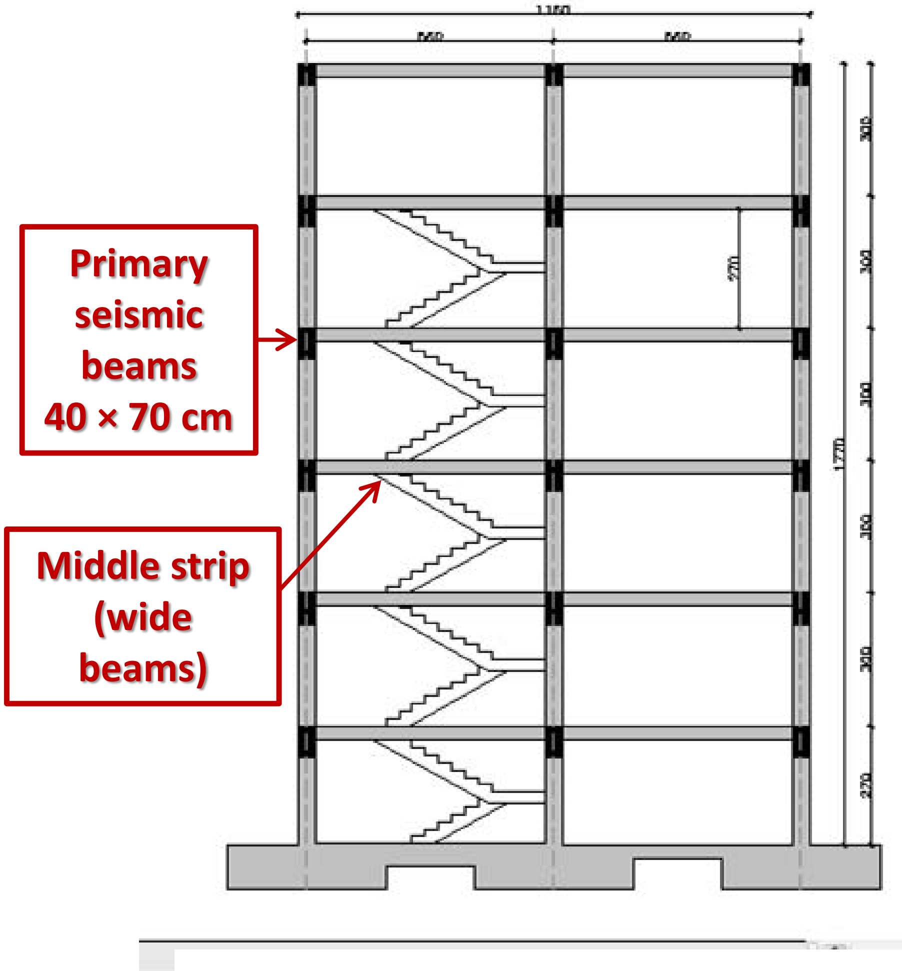


Case study: multi-storey RC frame

Y direction coupled walls

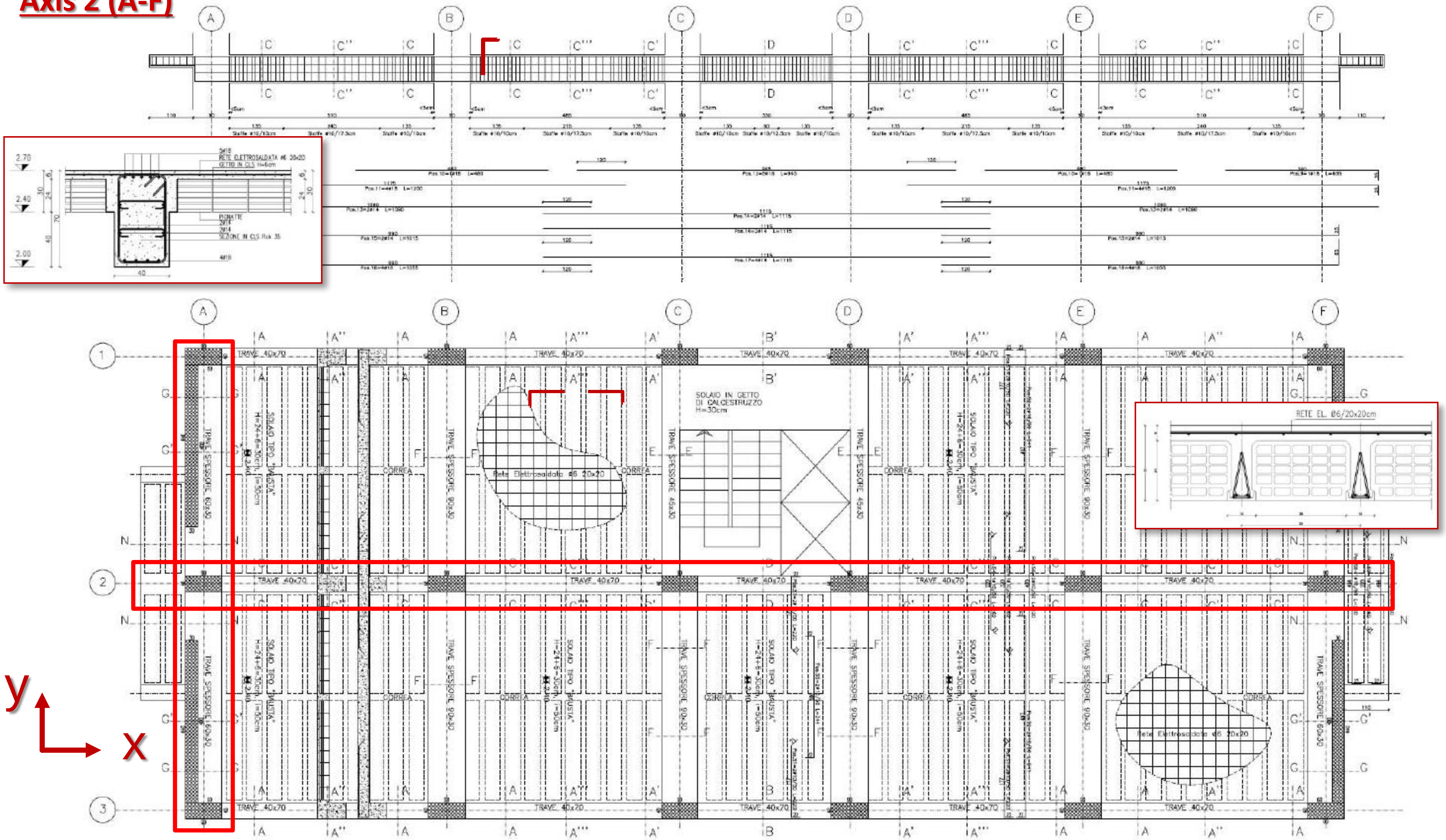


Y direction frames



Case study: multi-storey RC frame

Axis 2 (A-F)



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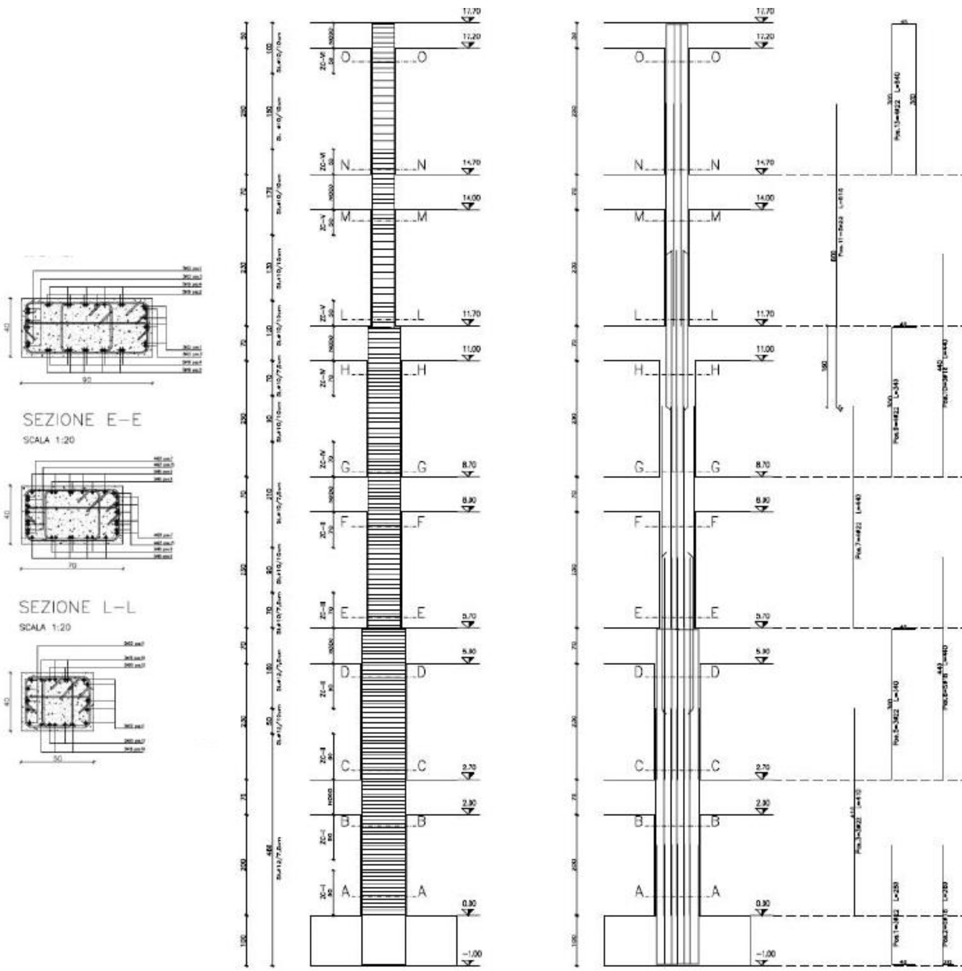
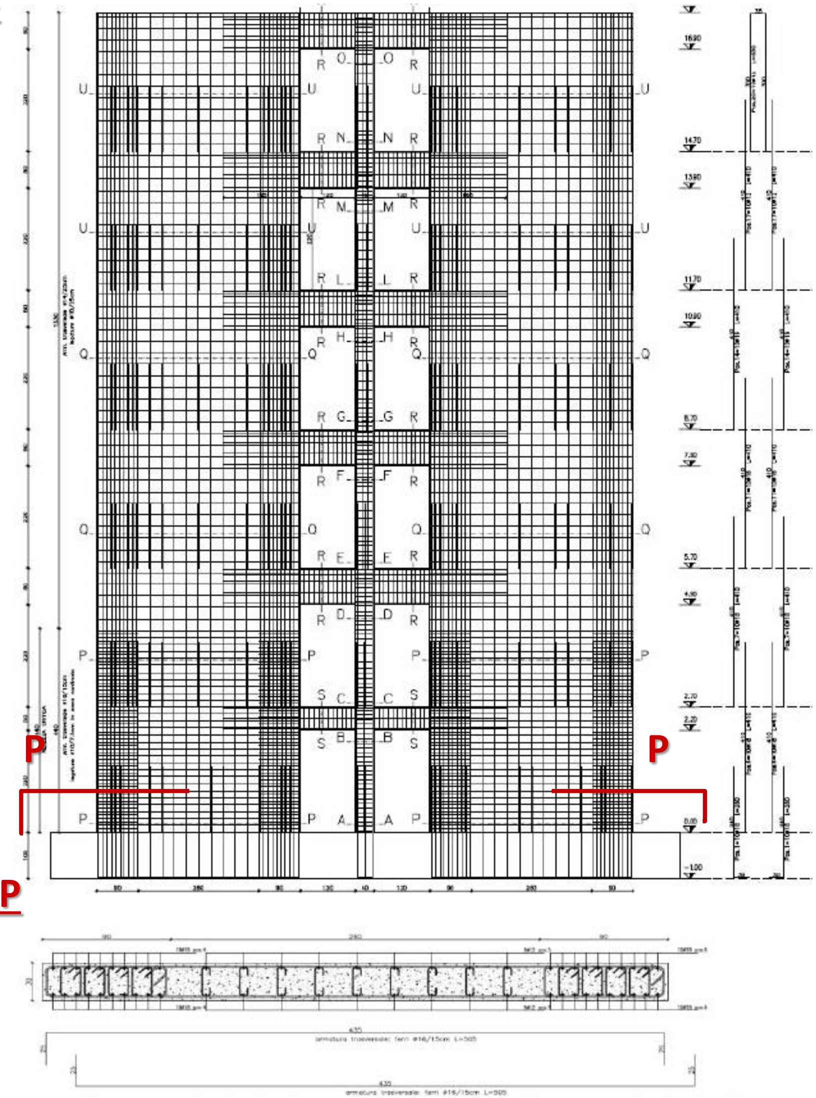
Case study: multi-storey RC frame

Column B2, C2, D2, F2



Wall W A13



Sec P-P

Comparison of Linear Analysis Methods

- The seismic effects and the effects of the other actions included in the seismic design situation may be determined on the basis of the linear-elastic behaviour of the structure. Both **types of linear-elastic analysis** have been performed in the present example:
- **lateral force method of analysis**
  - We applied this analysis on both 2D and 3D models even though the building is irregular in height; the 2D analysis has been performed to the sole purpose of comparison
- **modal response spectrum analysis**, which is applicable to all types of buildings
  - The **reference method for determining the seismic effects shall be the modal response spectrum analysis**, using a linear-elastic model of the structure and the design spectrum

*Moreover, a **non-linear static (pushover) analysis** has been also performed on the 2D model in X direction to assess ductility capacity*

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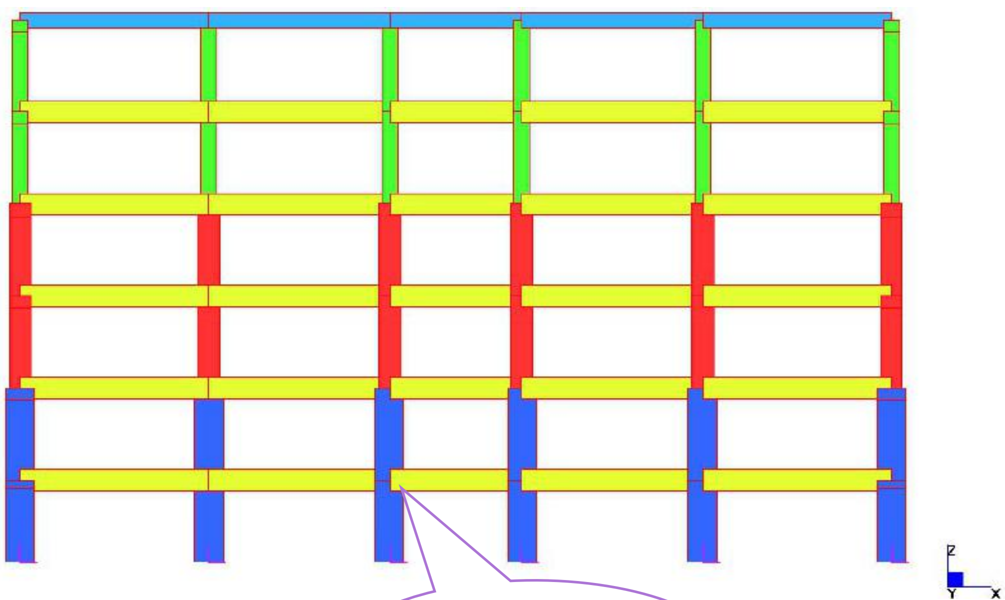
Lateral force method - 2-D Model

Seismic action effects approximated by horizontal forces increasing linearly along the height of the building (fundamental mode shape approximation of a regular building); material behaviour is linear.

FRAME SYSTEM

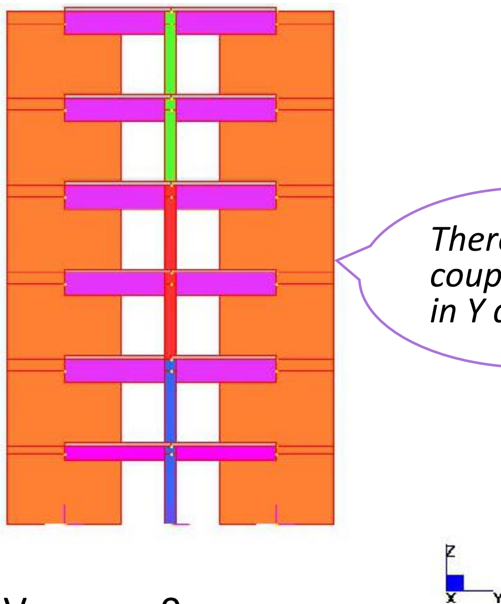


$V_{seismic,x} = F_{tot}/3$

$V_{seismic,y} = 0$

There are 3 2-D frames in X direction

WALL SYSTEM



There are 2 coupled walls in Y direction

$V_{seismic,x} = 0$

$V_{seismic,y} = F_{tot}/2$

Fundamental Period T_1

- For buildings with heights of up to 40 m the value of T_1 (s) may be approximated by the expression:

$$T_{1,x} = C_1 \cdot H^{3/4} = 0{,}075 \cdot 17{,}7^{3/4} = 0{,}628\ s$$

in X direction (RC frame)

- There are other formulas available in scientific literature and regulations; for example in Y direction the following expression for RC wall structures may be applied

$$T_{1,y} = 0{,}09 \cdot \frac{H}{L^{1/2}} = 0{,}09 \cdot \frac{17{,}7}{11{,}6^{1/2}} = 0{,}468\ s$$

  – L is the length of the building (wall), in m

- A behaviour factor $q=q_0K_R=(4.5\text{x}1.2)\times 0.8=4.32$ was adopted for calculations of the design spectral accelerations

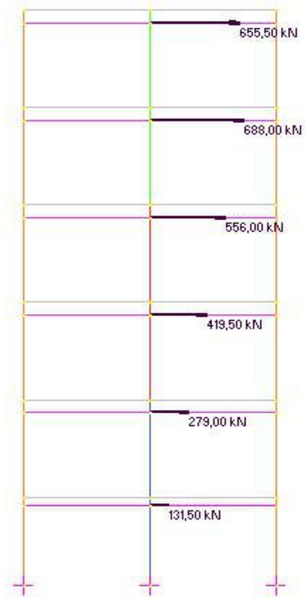
Lateral equivalent static forces

Lateral forces to be applied in X direction to RC frames

T_1	0.647	sec
$S_d(T_1)$	1.41	m/sec^2
λ	0.85	
g	9.81	m/sec^2
W	21643.28	kN
$F_b = S_d(T_1)W\lambda/g$	2644.187	kN

W1 [kN]	3712.80	z1 [m]	2.70	$F1,_{TELAIO}$	42.46
W2	3731.16	z2	5.70	F2	90.07
W3	3677.16	z3	8.70	F3	135.49
W4	3623.16	z4	11.70	F4	179.53
W5	3569.16	z5	14.70	F5	222.21
W6	2823.27	z6	17.70	F6	211.64

Permanent and variable vertical loads and lateral seismic forces applied to storeys, frames and walls.
For the sake of brevity only few distributions are represented here.



Lateral Forces to be applied in Y Direction(Coupled RC walls)

T_1	0.432	sec
$S_d(T_1)$	2.24	m/sec^2
λ	0.85	
g	9.81	m/sec^2
W	21643.28	kN
$F_b = S_d(T_1)W\lambda/g$	4200.695	kN

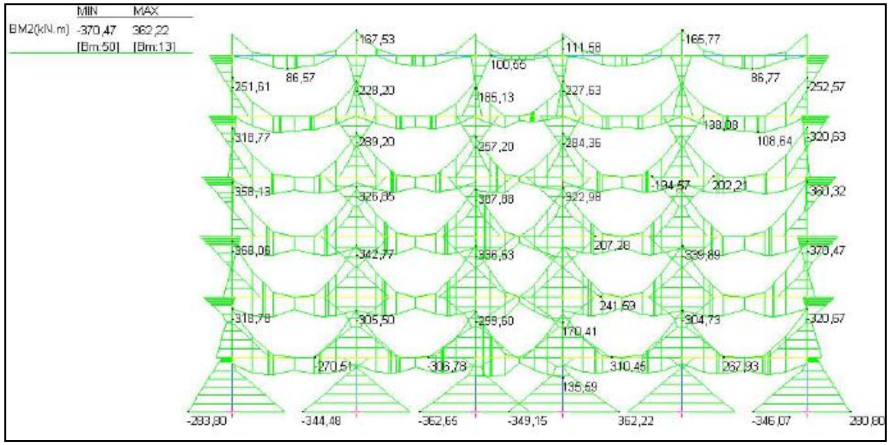
W1 [kN]	3712.80	z1 [m]	2.70	$F1,_{PARETE}$ [kN]	101.17
W2	3731.16	z2	5.70	F2	214.64
W3	3677.16	z3	8.70	F3	322.87
W4	3623.16	z4	11.70	F4	427.82
W5	3569.16	z5	14.70	F5	529.51
W6	2823.27	z6	17.70	F6	504.33

Torsional effects (RC walls)

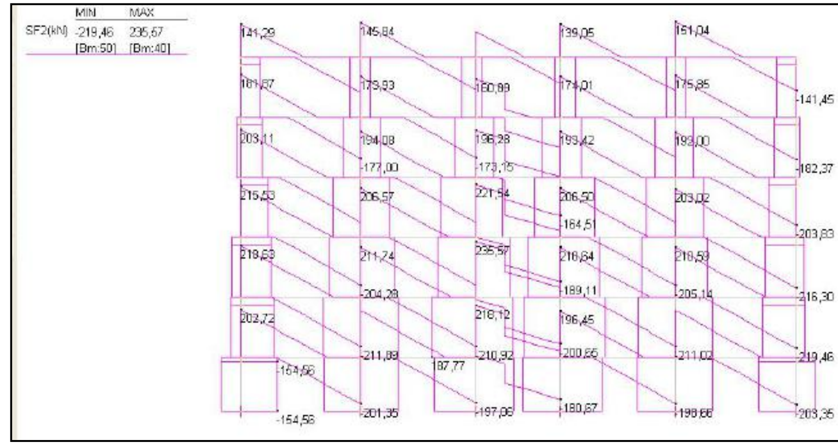
x		13.85
L_e		27.7
$\delta=1+0.6x/L_e$	amplification factor	1.3

Frame envelope results

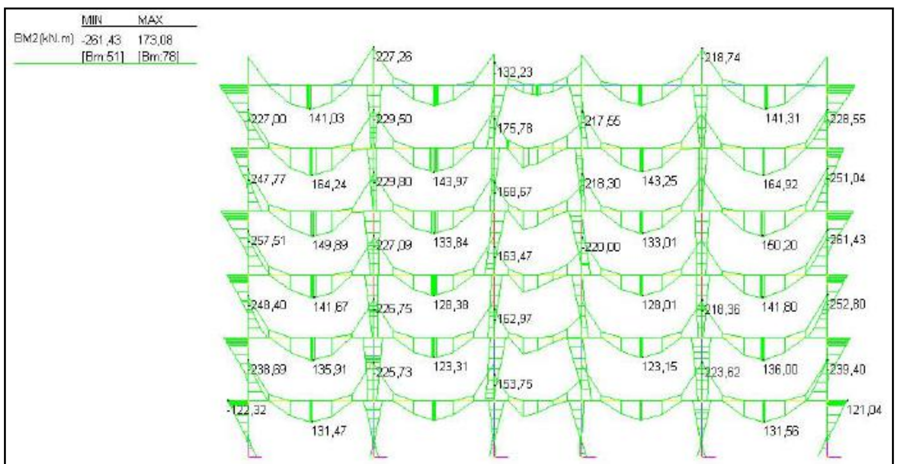
Seismic bending moments envelope



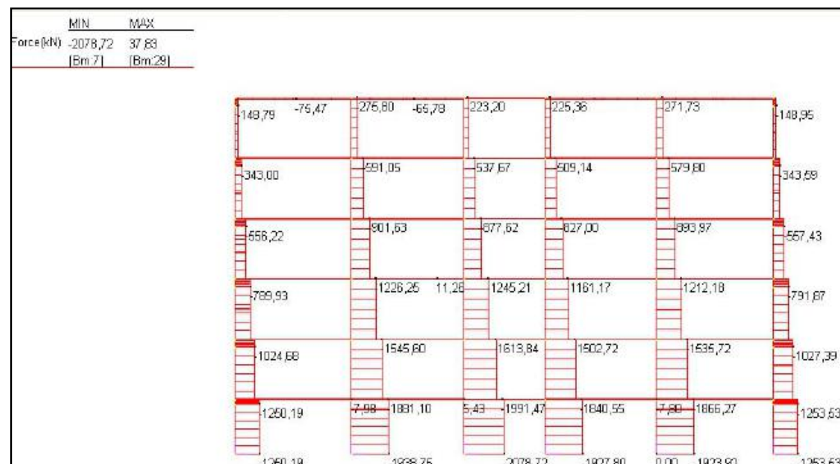
Seismic shear forces envelope



Static loads bending moments

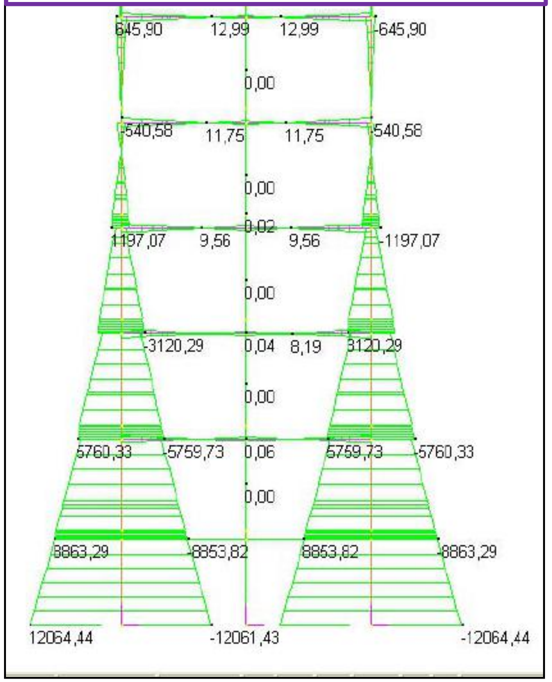


Static loads axial load forces

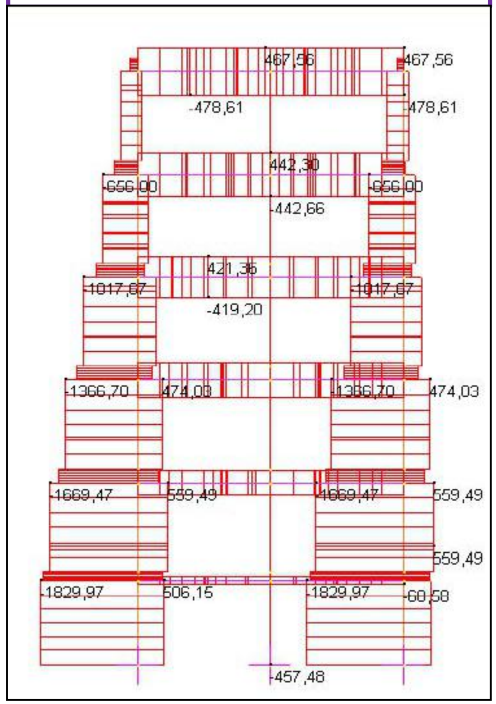


Wall envelope results

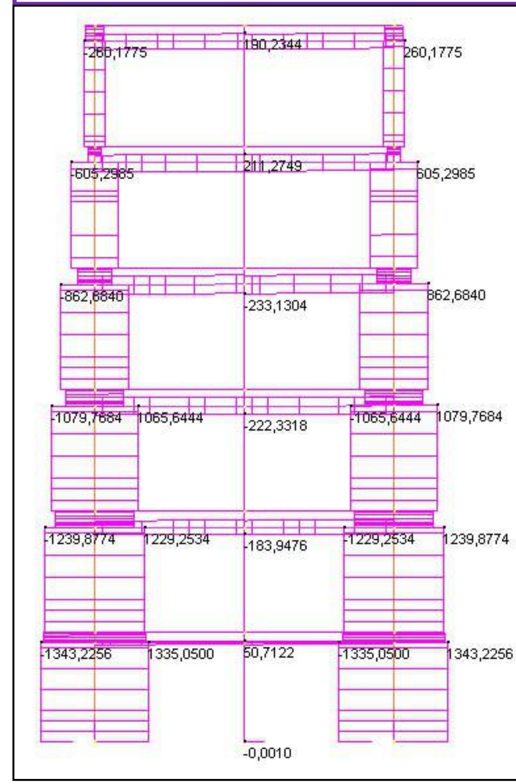
Seismic bending moments envelope

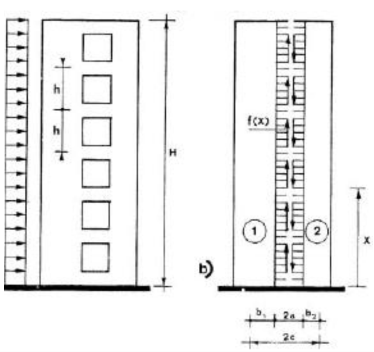


Seismic axial load envelope



Seismic shear forces envelope





$M_{tot,base}=M_1+M_2+N*2c$ where

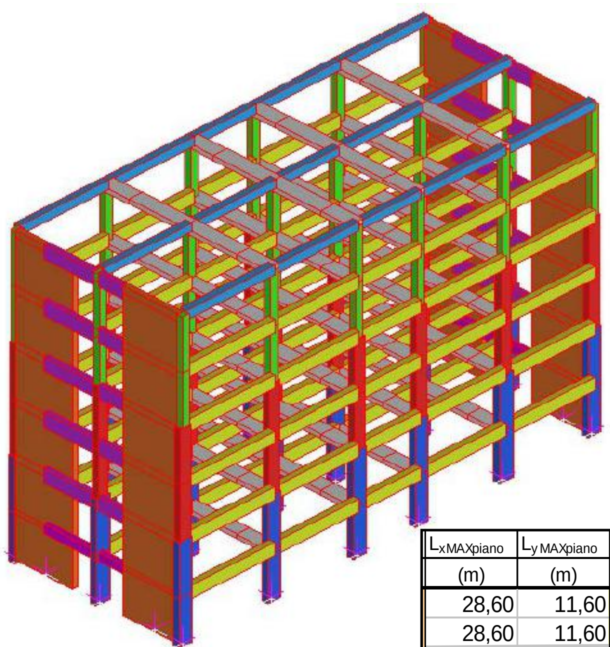
$N=\sum f(x)$ sum of shear forces on coupling beams of each level

Coupling condition, by its definition in EC8 §5.1.2, shall able to reduce the sum of the base bending moments by at least 25% of the single walls:

$N*2c > 25\%\ M_{tot,base}$ with 2c=7.30m

Lateral forces method: 3-D Model

- Beam elements, fixed ends at ground level, lateral forces equivalent to seismic action effects approximated by horizontal displacements increasing linearly along the height of the building (fundamental mode shape approximation of a regular building); material behaviour is linear and flexural behaviour is controlled by elastic member stiffness $(EJ)_i$
- Seismic action is applied at each floor centre of mass; torsional effects may be considered by means of equivalent bending moments $M_t = F_i \cdot e$

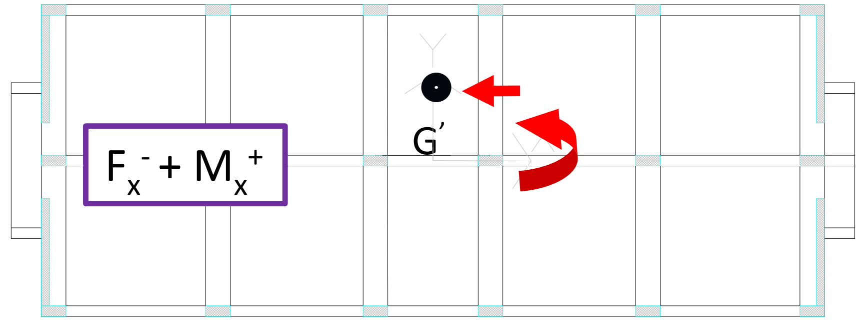


(§7.2.6 N.T.C): (…) centre of mass at each floor *i* shall be considered as being displaced from its nominal location in each direction by an accidental eccentricity:

$$e = 0.05\ L_{max}$$

Overall there will be eight combinations considering four displaced positions of G (e.g. G ', G'', G''', G^{IV}) and 2 seismic directions (X and Y). For example:

$L_{xMAXpiano}$	$L_{yMAXpiano}$	$E_{GX,Agg}$	$E_{GY,Agg}$
(m)	(m)	(m)	(m)
28,60	11,60	1,43	0,58
28,60	11,60	1,43	0,58
28,60	11,60	1,43	0,58
28,60	11,60	1,43	0,58
28,60	11,60	1,43	0,58
28,60	11,60	1,43	0,58



Lateral forces method: 3-D Model

In general the horizontal components of the seismic action shall be taken as acting simultaneously. The combination of the horizontal components of the seismic action may be accounted:

1. with **S.R.S.S. combination**

$$E_E^{\max} = \sqrt{E^2_{Ex} + E^2_{Ey}}$$

2. by using **both of the two following combinations**

- "+" implies "to be combined with'';
- E_{Ex} represents the action effects due to the application of the seismic action along the chosen horizontal axis *x* of the structure;
- E_{Ey} represents the action effects due to the application of the same seismic action along the orthogonal horizontal axis *y* of the structure.

$$E_{1-4} = \pm E_{Ex} \pm 0.3 E_{Ey}$$

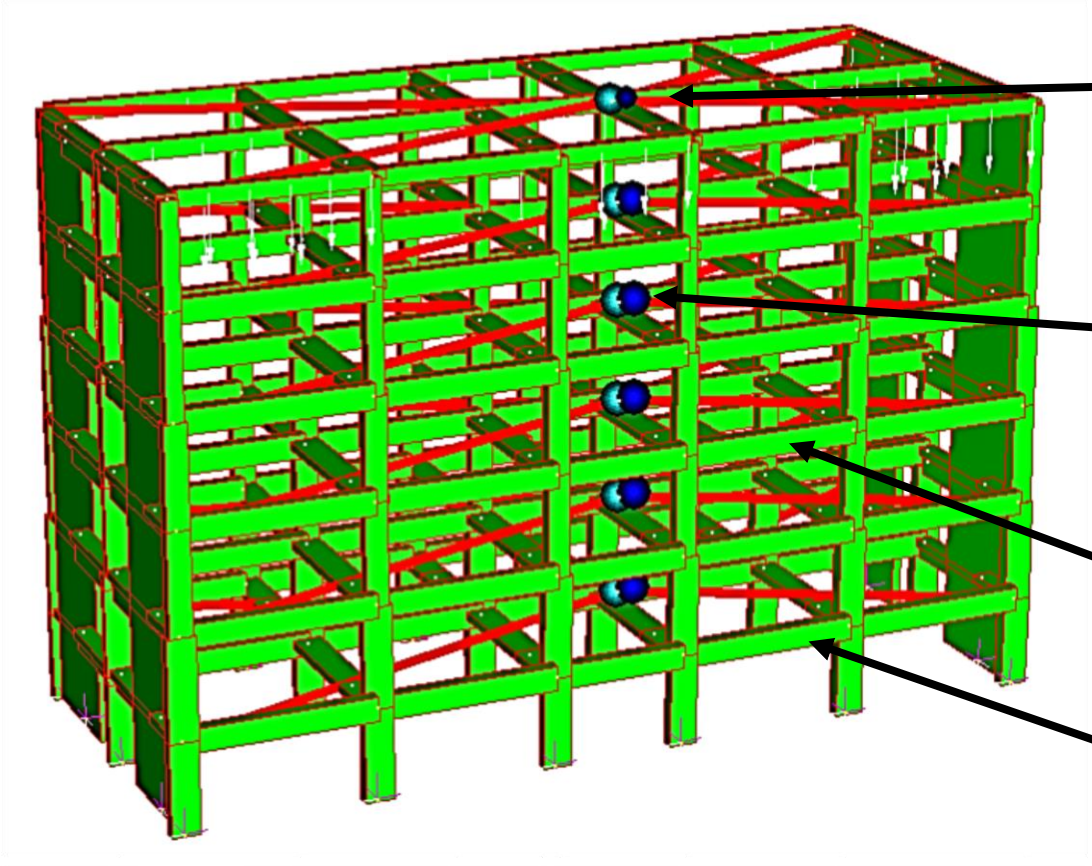
$$E_{5-8} = \pm E_{Ey} \pm 0.3 E_{Ex}$$

	Combinazione 9	Combinazione 10	Combinazione 11	Combinazione 12	Combinazione 13	Combinazione 14	Combinazione 15	Combinazione 16
	0.3(Ex+ Mx+)+(Ey+My+	**0.3(Ex- Mx+)+(Ey+My+**	**0.3(Ex- Mx-)+(Ey+My-)**	**0.3(Ex+ Mx-)+(Ey+My**	**0.3(Ex+ Mx+)+(Ey-My+**	**0.3(Ex- Mx+)+(Ey-My+**	**0.3(Ex+ Mx-)+(Ey-My-)**	**0.3(Ex- Mx-)+(Ey-My-)**
Ex+	0,3	0	0	0,3	0,3	0	0,3	0
Ex-	0	0,3	0,3	0	0	0,3	0	0,3
Ey+	1	1	1	1	0	0	0	0
Ey-	0	0	0	0	1	1	1	1
MEx+	0,3	0,3	0	0	0,3	0,3	0	0
MEx-	0	0	0,3	0,3	0	0	0,3	0,3
MEy+	1	1	0	0	1	1	0	0
MEy-	0	0	1	1	0	0	1	1
	Combinazione 1	Combinazione 2	Combinazione 3	Combinazione 4	Combinazione 5	Combinazione 6	Combinazione 7	Combinazione 8
	(Ex+Mx+)+0.3(Ey+My+)	**(Ex+Mx+)+0.3(Ey-My+)**	**(Ex- Mx-)+0.3(Ey+My-)**	**(Ex- Mx-)+0.3(Ey-My-)**	**(Ex+ Mx-)+0.3(Ey+My-)**	**(Ex+ Mx-)+0.3(Ey-My-)**	**(Ex- Mx+)+0.3(Ey-My+)**	**(Ex- Mx+)+0.3(Ey+My+)**
Ex+	1	1	0	0	1	1	0	0
Ex-	0	0	1	1	0	0	1	1
Ey+	0,3	0	0,3	0	0,3	0	0	0,3
Ey-	0	0,3	0	0,3	0	0,3	0,3	0
MEx+	1	1	0	0	0	0	1	1
MEx-	0	0	1	1	1	1	0	0
MEy+	0,3	0,3	0	0	0	0	0,3	0,3
MEy-	0	0	0,3	0,3	0,3	0,3	0	0

If, for generality, we ignore the symmetry about the Y-axis, we get 32 combinations (8 pairs of orthogonal actions E_{1-4} E_{5-8} combined with 4 positions of G (e.g. G ', G'', G''', G^{IV})

Response spectrum analysis

3-D MODEL



Rigid links to model rigid diaphragm behaviour of the floors

Translational and rotational masses placed in the centre of mass of each floor to account for non-modelled elements

$$I_p = M\rho^2\ (\mathrm{tm}^2), \quad \rho = \left(\sqrt{\frac{a^2+b^2}{12}}\right)$$

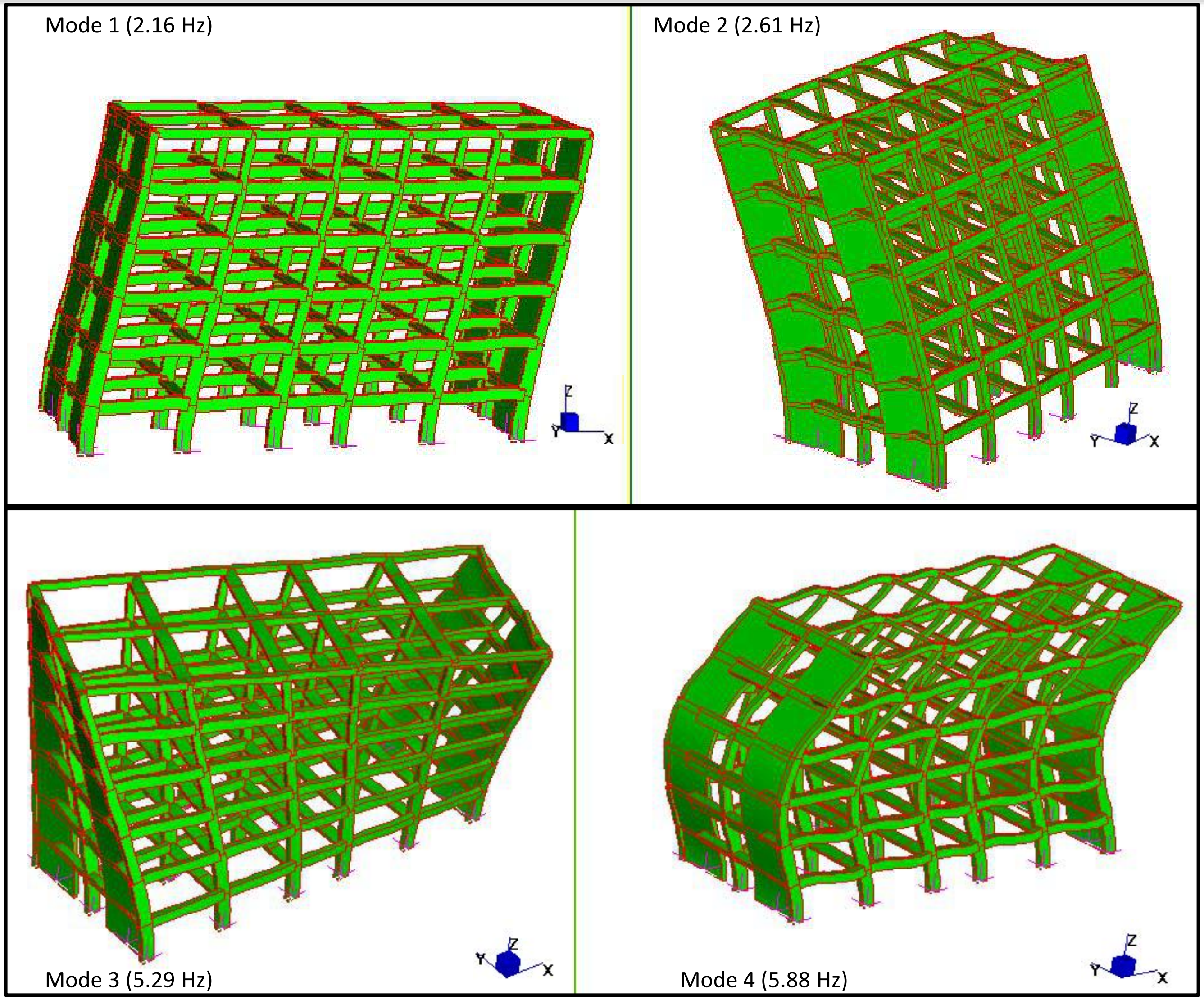
Stiffness properties of (cracked) elements

Beams: $(EJ)_{II}=0.5\ (EJ)_I$

Columns: $(EJ)_{II}=0.8\ (EJ)_I$

Option to use rigid beam-column joints

Response spectrum analysis



$$(\mathbf{K} - \omega_i^{\,2}\mathbf{M})\overline{\varphi}_i = \overline{0}$$

φ_i eigenvectors

Mode 1: Translational along X (1st deformed shape)

Mode 2: Translational along Y (1st deformed shape)

Mode 3: Torsional

Mode 4: Translational along Y (2nd deformed shape)

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Response spectrum analysis

- Eigenvalue problem $|K - \omega^2 M| = 0$
- Periods and frequencies of vibration are evaluated
- For each *ith* mode of vibration, generalized mass, effective modal mass and modal participation factor are evaluated

- EC8 requirement:
  - the sum of the effective modal masses for the modes taken into account amounts to at least 85% of the total mass of the structure;
  - all modes with effective modal masses greater than 5% of the total mass are taken into account.

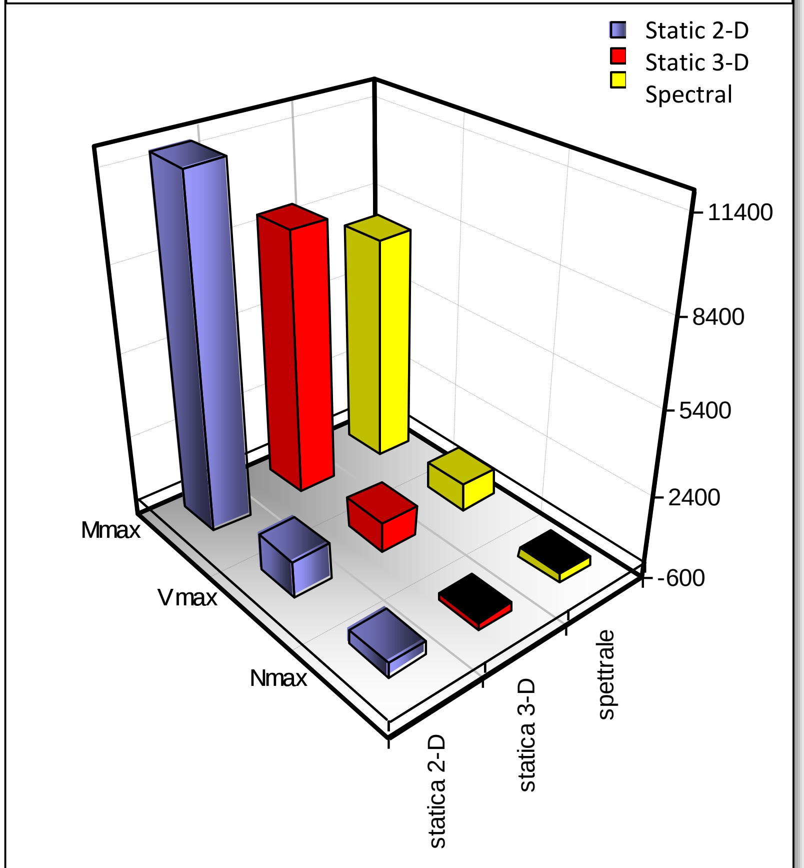
N_{floors}x3= 6x3=18 DoF

MODE PARTICIPATION

Mode	Frequency (Hz)	Modal Mass (Engineering)	PF-X (%)	PF-Y (%)	PF-Z (%)
1	2.157E+00	8.054E+05	74.626	0.000	0.000
2	2.615E+00	7.074E+05	0.000	69.101	0.000
3	5.289E+00	1.820E+05	0.073	0.000	0.000
4	5.882E+00	7.616E+05	12.153	0.000	0.000
5	1.069E+01	1.142E+06	5.221	0.000	0.000
6	1.145E+01	1.043E+06	0.000	19.679	0.000
7	1.527E+01	9.559E+05	3.092	0.000	0.000
8	2.238E+01	2.289E+05	0.008	0.000	0.000
9	2.245E+01	1.167E+04	0.000	0.001	0.000
10	2.278E+01	7.809E+05	2.410	0.000	0.000
11	2.282E+01	3.599E+04	0.000	0.000	33.447
12	2.312E+01	9.115E+03	0.000	0.000	0.000
13	2.332E+01	2.421E+04	0.004	0.000	0.000
14	2.374E+01	1.072E+04	0.000	0.000	0.000
15	2.424E+01	3.030E+04	0.000	0.000	0.871
16	2.451E+01	8.459E+03	0.000	0.000	0.000
17	2.476E+01	1.512E+04	0.000	0.004	0.000
18	2.525E+01	1.031E+04	0.000	0.000	0.000
19	2.557E+01	1.805E+04	0.000	0.010	0.000
20	2.558E+01	1.527E+04	0.001	0.000	0.000
21	2.585E+01	1.473E+04	0.000	0.002	0.000
22	2.596E+01	8.084E+05	0.000	6.744	0.000
TOTAL MASS PARTICIPATION FACTORS			97.587	95.593	37.194

Linear static and spectral analysis comparison

(M, V, N) at the base of the RC Walls

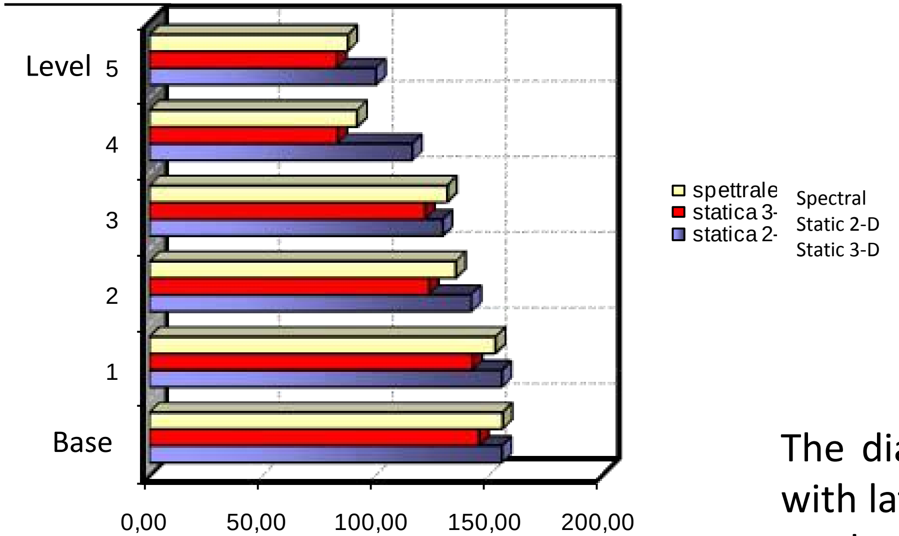


Walls	M e N alla base della parete			
		statica 2-D	statica 3-D	spettrale
	Mmax (kNm)	12064,6	9275,53	7998,7
	Vmax (kN)	1343,23	1090,21	959,82
	Nmax (kN)	506,18	-296,08	-363,12
Beams	M trave Ef,e (PIANO2)			
		statica 2-D	statica 3-D	spettrale
	My,max(kNm)	86,51	75,13	69,47
	My,min(kNm)	-330,69	-332,81	-407,22
	M trave Ef,f (PIANO2)			
		statica 2-D	statica 3-D	spettrale
	My,max(kNm)	75,21	62,55	83,98
	My,min(kNm)	-370,45	-371,01	-392,71
	M trave CD,d (PIANO2)			
		analisi statica 2-D	analisi statica 3-D	spettrale
	My,max(kNm)	170,40	165,79	115,58
	My,min(kNm)	-336,53	-334,85	-438,81
Columns	M e N alla base del pilastro D			
		statica 2-D	statica 3-D	spettrale
	$M_{y,max}$	1599,37	1602,84	1580,08
	Vmax	179,11	168,46	183,22
	N_{max}	-362,65	-345,65	-386,42

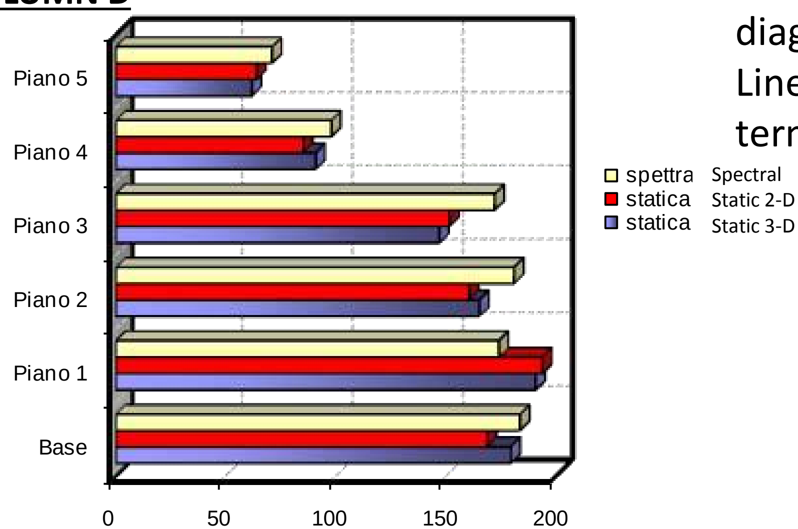
Linear static analyses on the two 2-D models (in X and Y directions) show considerably conservative internal forces (M, V) on RC walls. **Amplification factor δ for accidental torsional effects is overestimated.**

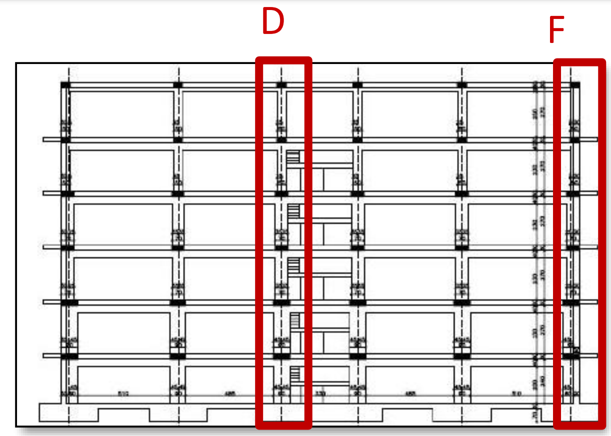
Shear forces on columns

COLUMN F



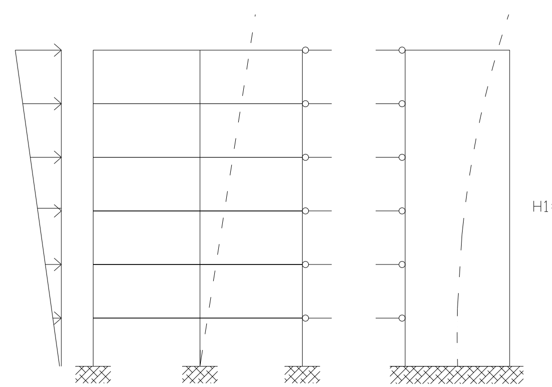
COLUMN D



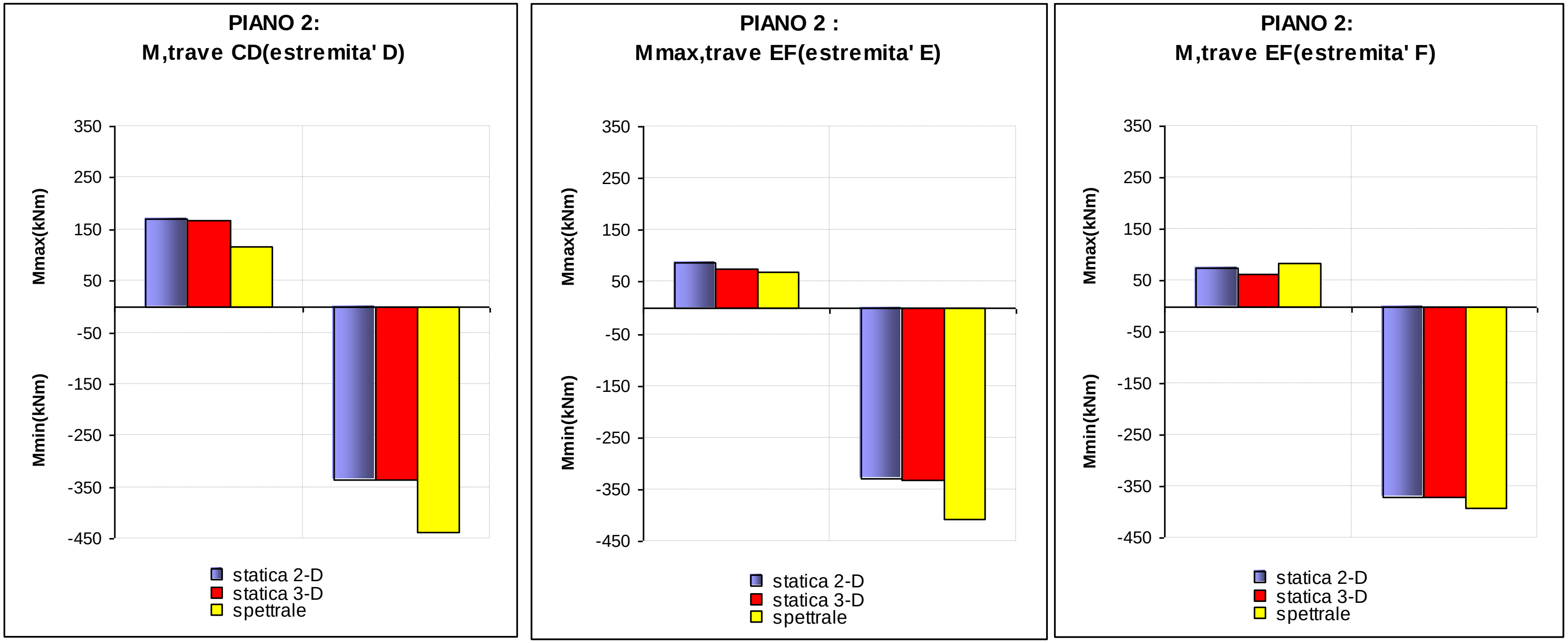


The diagram of shear forces due to seismic action obtained with lateral force method of analysis is related only to the first mode of vibration, thus it is significantly different from the diagram given by a multi-modal spectral analysis.

Linear static analysis isn't always the most conservative in terms of safety.



Internal Forces on beams



Concerning beams, lateral force method of analysis (linear static analysis) is not always the most conservative for both positive and negative bending moments. Internal forces obtained with lateral force method of analysis on both planar (2-D) and spatial (3-D) models tend to be more consistent between each other than those obtained with modal response spectrum analysis.

- 2. NON-LINEAR ANALYSES

Non-Linear Analyses

- Nonlinear Analysis is <u>harder:</u>
  - It requires much more thought when setting up the model
  - It requires more thought when setting up the analysis
  - It takes more computational time
  - It does not always converge
- <u>BUT Many Problems Require Nonlinear Analysis</u>
- **<u>Geometric Nonlinearities</u>** - occur in model when applied load causes large displacement and/or rotation, large strain, or a combination of both
- **<u>Material nonlinearities</u>** - Structural concrete is an inherently nonlinear material both at strength limit state and service loads
- **Contact nonlinearities**

- Linear analysis is not adequate and nonlinear analysis is <u>necessary</u> when
  - Designing high performance components
  - Establishing the causes of failure (PBD design)
  - Simulating true material behavior
  - Trying to gain a better understanding of physical phenomena

Material non-linearity and modelling

1D with lumped plasticity

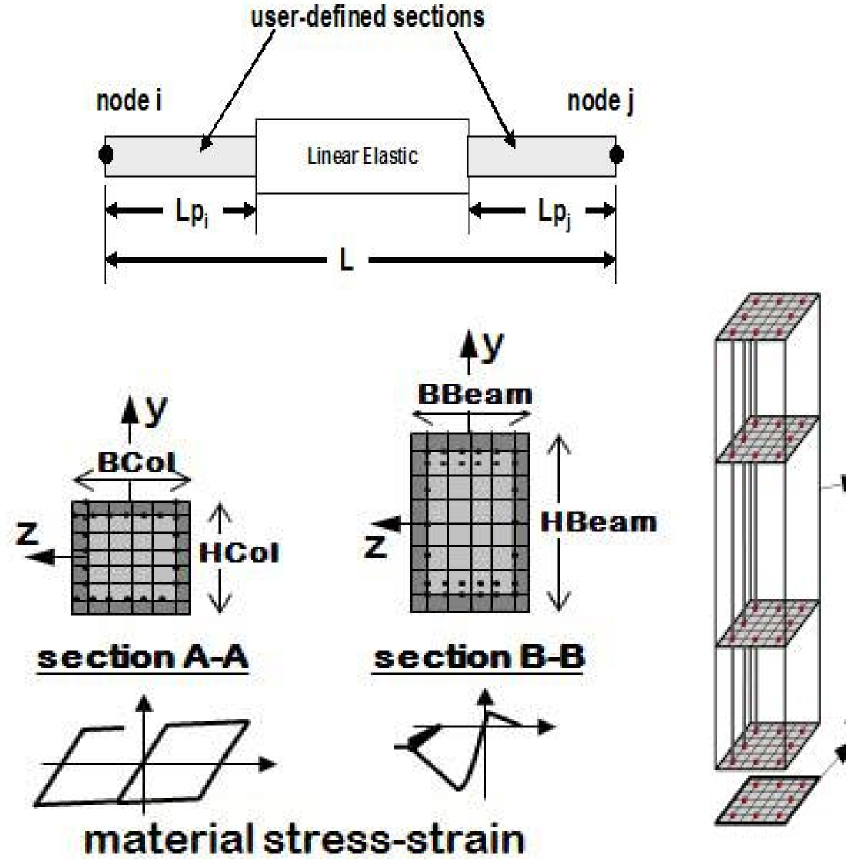
- Moment curvature diagrams
- Force displacement diagrams

1D with distributed plasticity

- Nonlinear material uniaxial fibers
- Fiber based elements

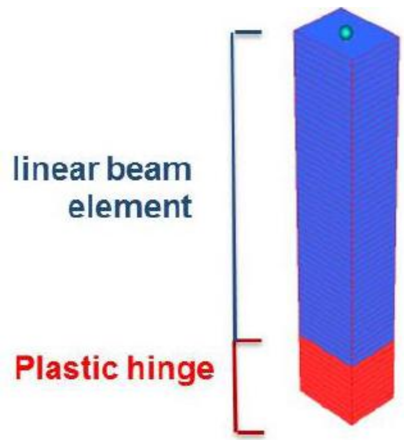
2D & 3D FEM models

- Micro-models
- 2D & 3D nonlinear constitutive relationships



PLASTIC HINGE METHOD

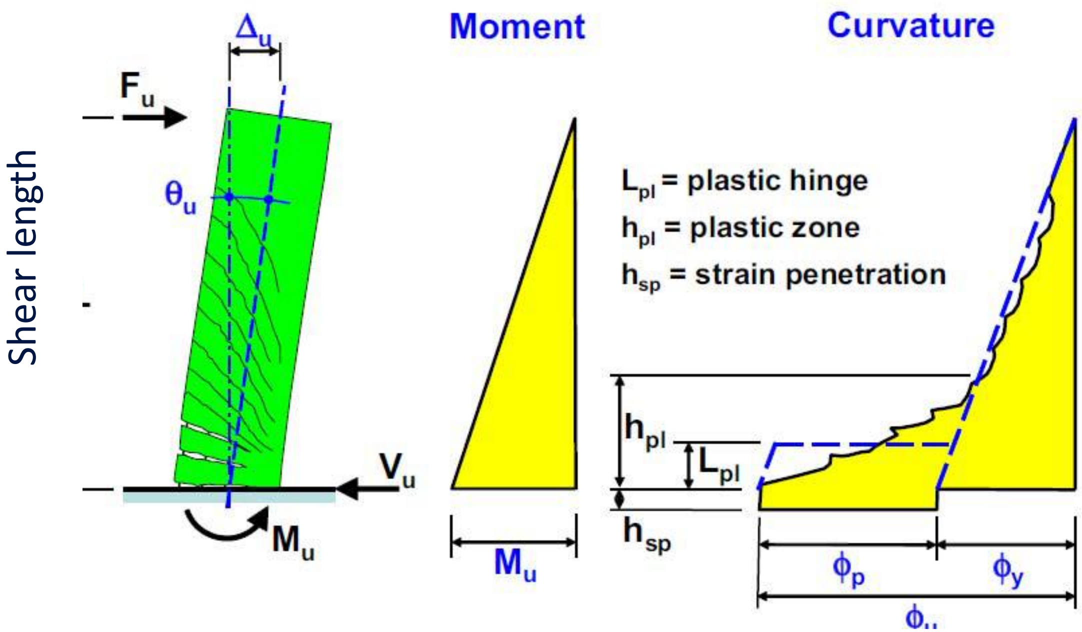
- It consists on replacing the real distribution of curvature with an equivalent distribution which can be easily integrated and quickly provide the deflected shape



$$L_{sp} = 0.022 f_{ye} d_{bl}$$

$$L_p = k L_c + L_{sp} \geq 2 L_{sp}$$

$$k = 0.2 \left(\frac{f_u}{f_y} - 1 \right) \leq 0.08$$



$$\boxed{\Delta_y = \frac{\phi_y (H + L_{sp})^2}{3}}$$

$$\Delta_p = \phi_p L_p L_c$$

$$\Delta_p = (\phi_u - \phi_y) L_p \left(H - \frac{L_p}{2} \right)$$

Example of M-C analysis

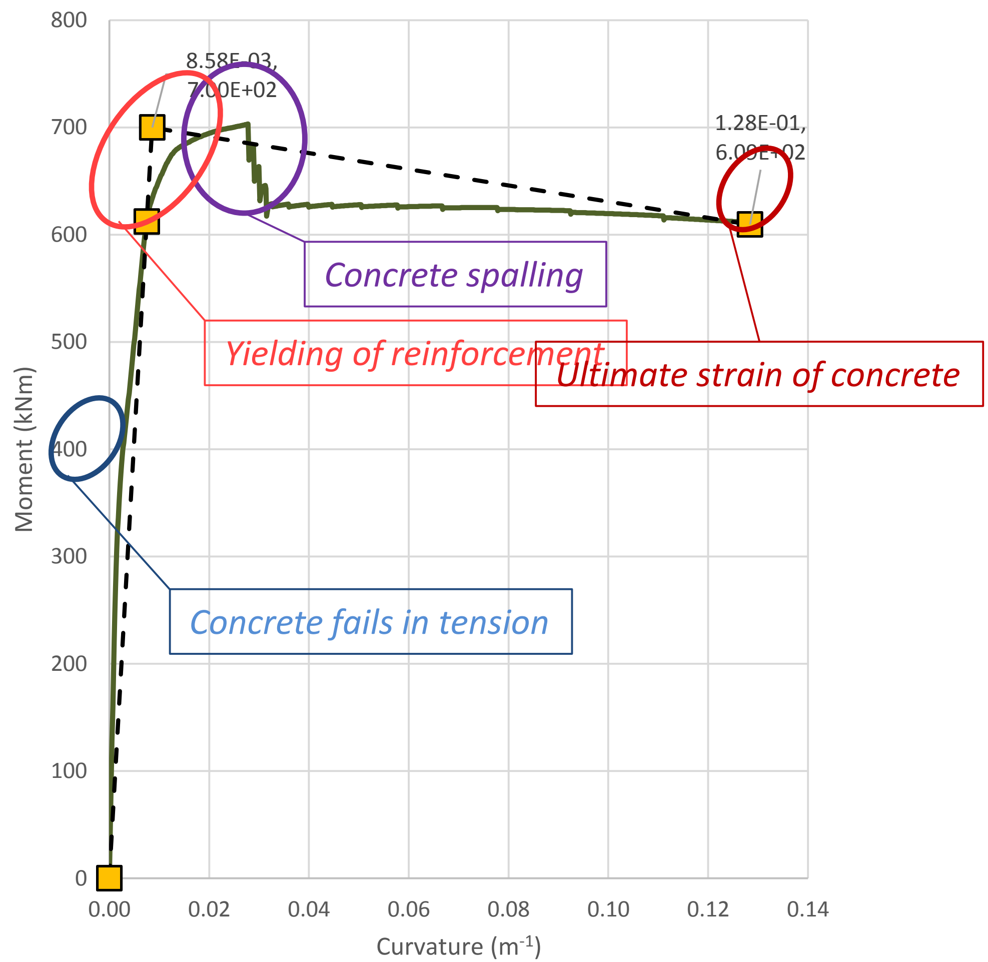
Square R.C. section geometry and materials

B = H = 60cm

f_c = 25 MPa

Steel grade: B450C

- Longitudinal steel ratio = 0.01 (i.e. 12Ø20)
- Transverse steel ratio = 0.003
- Normalized axial load = 0.15



- Nonlinear section response can be used to define a lumped plasticity element
- **Section ductility** can be evaluated

$$\mu_\varphi = \frac{\varphi_u}{\varphi_y} = \frac{1,28E-01}{8.53E-3} \approx 15$$

Plastic hinge length

Various researchers have proposed expressions for the plastic hinge length, which are calibrated from experimental data

There are many **parameters that affect the plastic hinge length**, however not all researchers agree on the significance of each. These parameters include

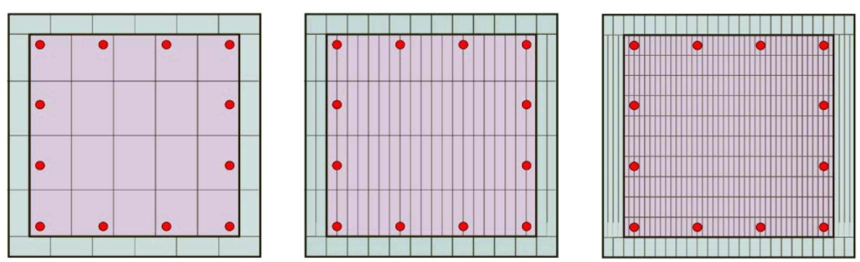
- moment gradient (column length)
- amount of reinforcement (reinforcement ratio)
- axial load level
- materials strength, such as steel yield strength (f_y) and concrete compressive strength (f'_c)
- aspect ratio

Corley (1966)	$L_p = 0.50D$
Priestley et al. (1996)	$L_p = 0.08L + 0.022 f_{ye} d_{bl} \geq 0.044 f_{ye} d_{bl}$ (MPa)
Priestley, Calvi and Kowalsky (2007)	$L_p = kL + 0.022 f_{ye} d_{bl} \geq 0.044 f_{ye} d_{bl}$ (MPa) *where* $k = 0.2\left(\frac{f_u}{f_y} - 1\right) \leq 0.08$
Berry et al (2008)	$L_p = 0.0375L + 0.01 f_y \frac{d_b}{\sqrt{f'_c}}$ *(psi)*
Bae & Bayrak (2008)	$L_p = L\left(0.5\frac{P}{P_0} + 3\frac{A_s}{A_g} - 0.1\right) + 0.25h + L_{sp} \geq 0.25h$ *where* $P_0 = 0.85 f'_c (A_g$

- Moment-Curvature analysis of a r.c. section – fiber section model

- **Materials models**
  - Uniaxial material models are assigned to each fiber of the section
  - Three materials are defined
    - *Core concrete (confined)*
    - *Cover concrete (unconfined)*
    - *Reinforcement steel*

- **Number of fibers in a section**
  - Aspects to consider: accuracy of results, computational cost, convergence problems
  - Consider the specific problem to be solved!
  - Use 1 fiber for each reinforcement bar
  - Do some trials for concrete discretization



Materials Used For NL Analyses

- CLS-CONCRETE 02 (KENT AND PARK, f'_{cc} calibrated on MANDER MODEL)

f'_c

$$f_l = \frac{2f_{yh}A_h}{D's} = 0.5\rho_v f_{yh}$$ lateral conf. stress

$$f'_{cc} = f'_c\left(2.254\sqrt{1+\frac{7.94f_l}{f'_c}} - 2\frac{f_l}{f'_c} - 1.254\right)$$

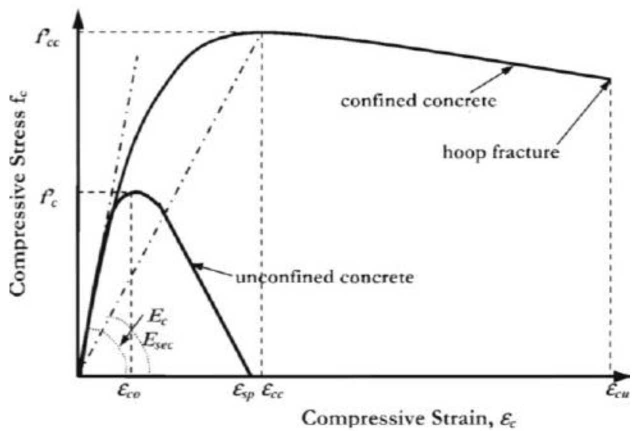
$$\varepsilon_{cc} = 0.002\left(1-5\left(\frac{f'_{cc}}{f'_c}-1\right)\right)$$

$$E_{\text{sec}} = f'_{cc}/\varepsilon_{cc} \quad f_c = \frac{f'_{cc}x\cdot r}{r-1+x^r}$$

$$E_c = 5000\sqrt{f'_c}$$

$$r = \frac{E_c}{E_c - E_{\text{sec}}}$$

$$x = \varepsilon_c/\varepsilon_{cc}$$



- REINFORC. STEEL – MENEGOTTO_PINTO MODEL

$$f_{ye} = 1.1f_y$$

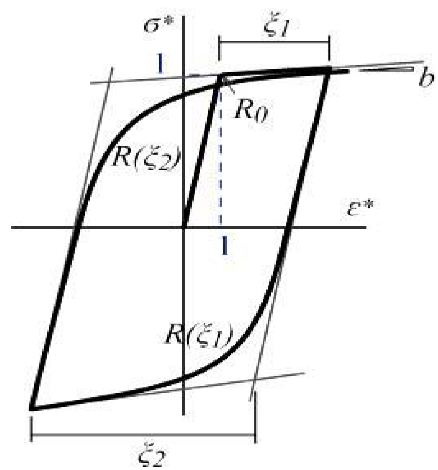
$$\sigma^* = b\varepsilon^* + (1-b)\varepsilon^*/\left(1+\varepsilon^{*R}\right)^{1/R}$$

$$\varepsilon^* = \frac{(\varepsilon-\varepsilon_r)}{(\varepsilon_0-\varepsilon_r)}$$

$$\sigma^* = (\sigma-\sigma_r)/(\sigma_0-\sigma_r)$$

$$R(\xi) = R_0 - a_1\xi/(a_2+\xi)$$

ε_r and σ_r are respectively the strain and tension at last inversion point, and are respectively the strain and tension at asymptotes intersection.



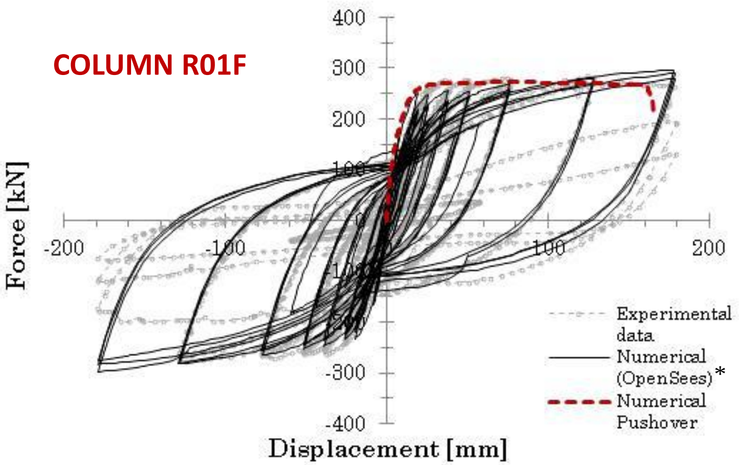
- Limit states defined for materials strain limits

- **LS1 – end of “elastic” phase**
  - First yielding of reinforcement steel
  - cracking of concrete cover
- **LS2 – damage limitation**
  - spalling of cover concrete
  - development of cracks withs greater than 1mm
- **LS3 – ultimate**
  - core concrete crushing
  - steel ultimate strain

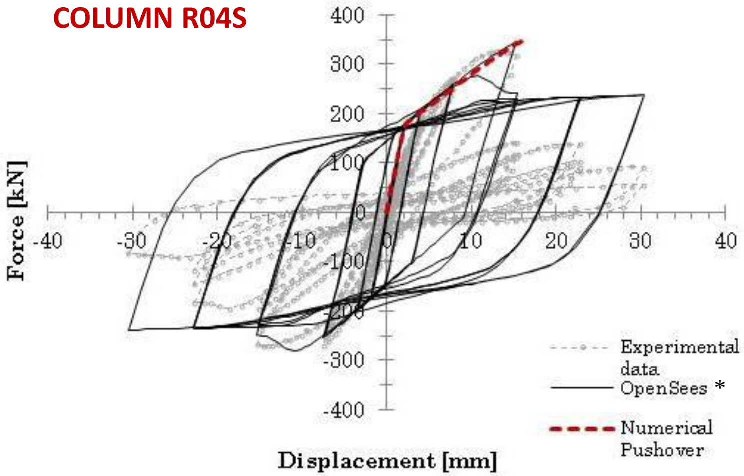
Materials	LS1	LS2	LS3
Concrete ε_c	0.002	0.004	ε_{cu}
Steel ε_s	$\varepsilon_y = \frac{f_y}{E_s}$	0.015	$0.6\varepsilon_{su}$

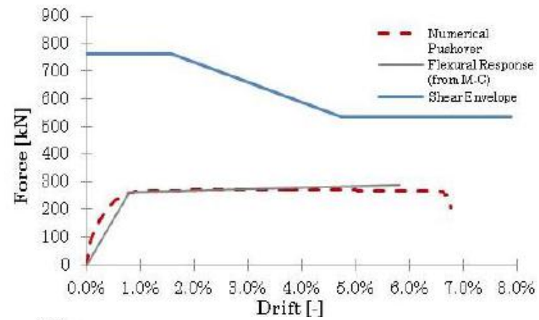
Aggregation of non-linear shear behaviour

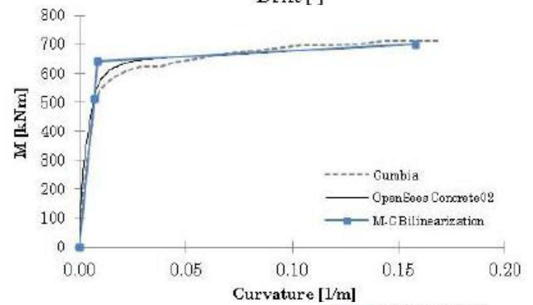
COLUMN R01F

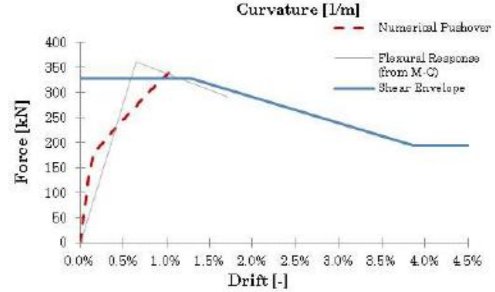


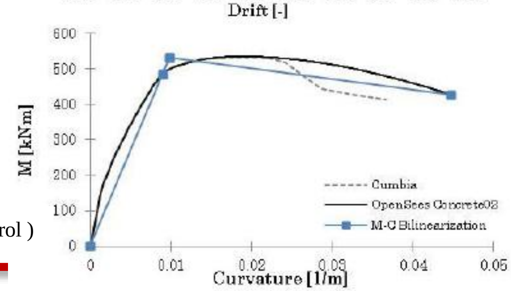
COLUMN R04S











* (Displacement control)

EXPERIMENTAL DATABASE (SPD – PEER)

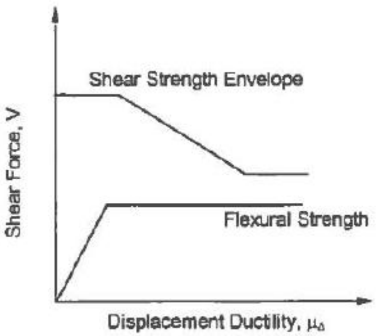
Database ID	Source	Authors	Section Type	Failure type
C01F	SPD-PEER	Lehman et al. (1998)	Circ.	Flex.
C02F	SPD-PEER	NIST	Circ.	Flex.
R01F	SPD-PEER	Park and Paulay (1990)	Rect.	Flex.
R02S	SPD-PEER	Imai and Yamamoto (1986)	Rect.	Shear
R03S	SPD-PEER	Lynn et al. (1998)	Rect.	Shear
R04S	SPD-PEER	Lynn et al. (1998)	Rect.	Shear
R05S	SPD-PEER	Lynn et al. (1998)	Rect.	Shear

		Geometric properties			Reinforcement ratio		
Database ID		*d (mm)*	*s (mm)*	*a/d*	ρ_l (%)		ρ_s (%)
C01F		609,6	31.75	4,00	1.49		0.698
C02F		1520	88.9	6.01	1.99		0.630
	b (mm)	*h (mm)*	*s (mm)*	*a/b*	ρ_l (%)	ρ_x (%)	ρ_y (%)
R01F	400	600	80/160	1.65	1.88	1.25	1.05
R02S	400	500	100	3.22	2.66	0.40	0.31
R03S	457	457	457	3.22	3.03	0.08	0.08
R04S	457	457	457	3.22	3.03	0.08	0.08
R05S	457	457	305	3.22	3.03	0.21	0.21

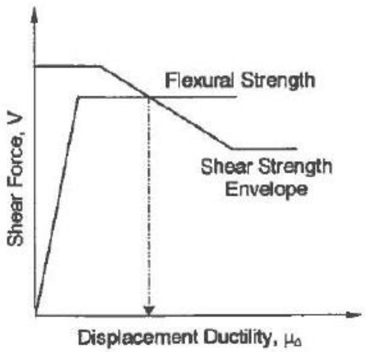
Aggregation of non-linear shear behaviour

Phase	Flexure	Shear	Total Stiffness
Phase I	K_{ff}	K_{sf}	$1/(1/K_{ff}+1/K_{sf})$
Phase II	K_{ff}	K_{ss}	$1/(1/K_{ff}+1/K_{ss})$
Phase III	K_{fy}	K_{ss}	$1/(1/K_{fy}+1/K_{ss})$

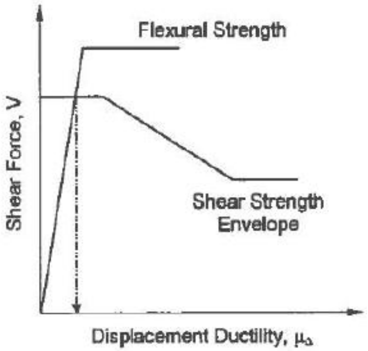
PIER FAILURE MODES (ACCORDING TO THE ATC-6)



Flexure (F)



Flexure –Shear (FS)



Brittle Shear (S)

SHEAR CRACKING EXPRESSIONS

General form $V_{cw} = f(v_{cw} A_w)$

ACI 318-02 $V_{cw} = \begin{cases} (0.29\sqrt{f'c} + 0.3 f_{pc}) b_w d \\ \le 0.41\sqrt{f'_c} b_w d \end{cases}$

(M.C.P.P., 2005) $V_{cr} = 0.215 \left(\frac{s}{d}\right)^{-0.57} (v_{cr} A_w)$

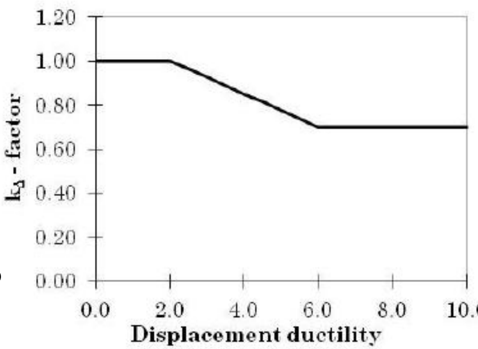
"Assessment purposes" $A_w = b_w d \quad v_{cr} = 0.5\sqrt{f'_c}\sqrt{1 + \frac{P}{0.5\sqrt{f'_c}A_g}}$

SHEAR ENVELOPE

Sezen e Moehle (2004)

$$V_n = k_\Delta (V_c + V_s)$$

$$k_\Delta = \begin{cases} 1{,}0 & \mu_\Delta \le 2 \\ 1 - 0.3 \cdot \frac{\mu_\Delta - 2}{4} & 2 < \mu_\Delta < 6 \\ 0{,}7 & \mu_\Delta > 6 \end{cases}$$

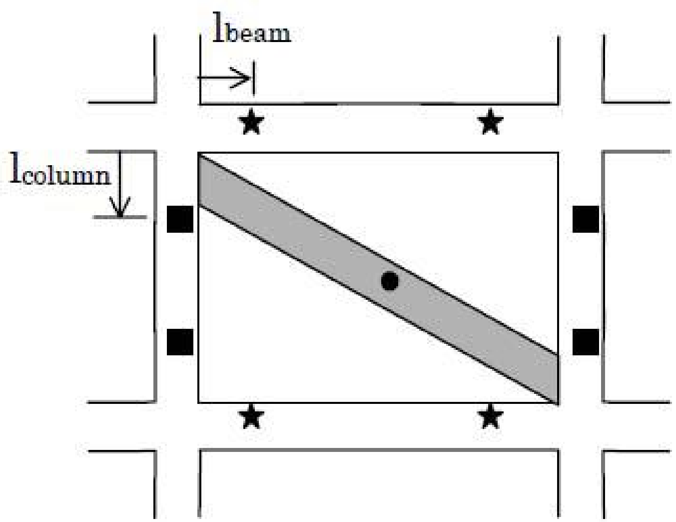


$$V_s = \frac{A_{sh} f_{yh} d}{s} \quad V_c = \frac{0.5\sqrt{f'_{c0}}}{a/d}\sqrt{1 + \frac{P}{0.5\sqrt{f'_{c0}}A_g}}(A_e) \quad 2 \le a/d \le 4$$

Infills-frame interaction

PUSHOVER ANALYSIS

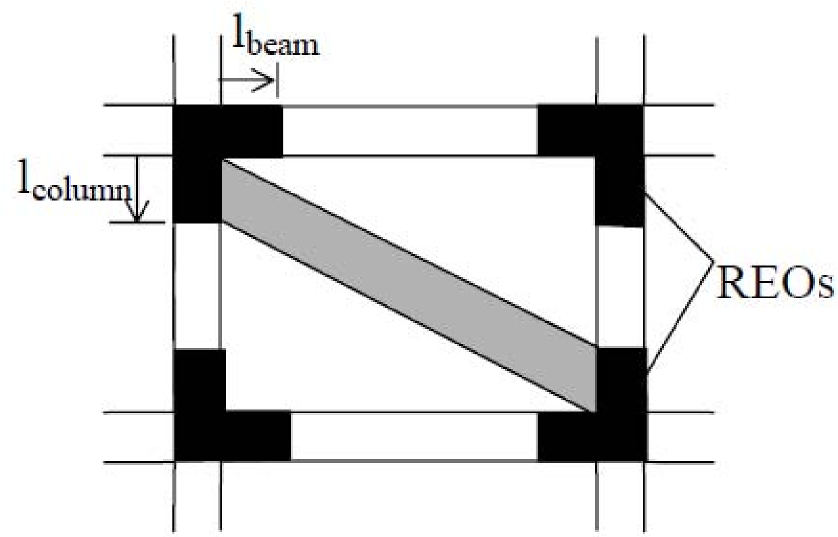
Plastic hinge placement



■ Axial-Moment and Shear Hinge

★ Moment and Shear Hinge

● Axial Hinge Only

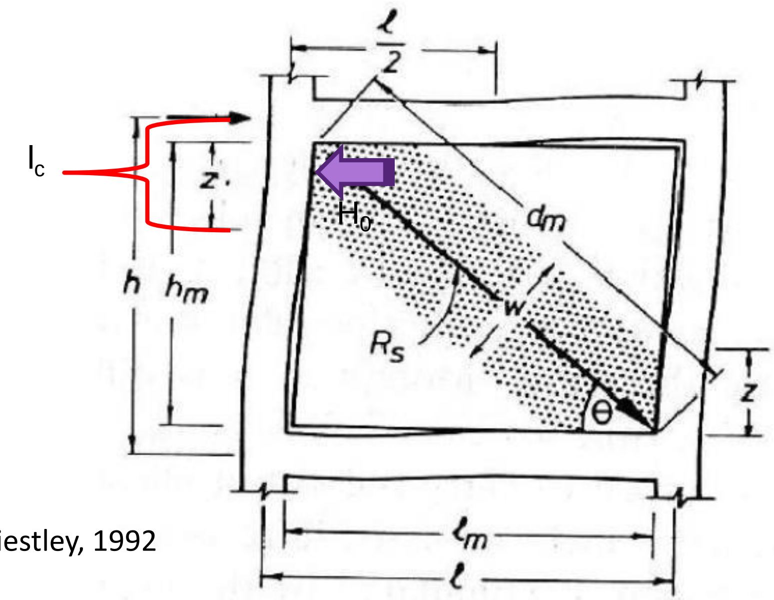


Rigid end-offset placement

Al-Chaar, 2002

LOCAL EFFECTS PRODUCED BY INFILL WALLS ON THE RC FRAME

- Equivalent strut produces shear on the adjacent columns along the contact lenght l_c
- This part of columns needs to be verified for $\boldsymbol{H_0}$ which is the minimum value of shear between the following ones (EC8 - §5.9(4)):
  a) shear horizontal strength of infill wall
  b) shear from capacity design procedure acting along the lenght l_c of the column



$$H_0 = min\left\{V_t\,;\,\gamma_{Rd}\cdot\frac{2\cdot M_{C,Rd}}{l_c}\right\}$$

Paulay and Priestley, 1992

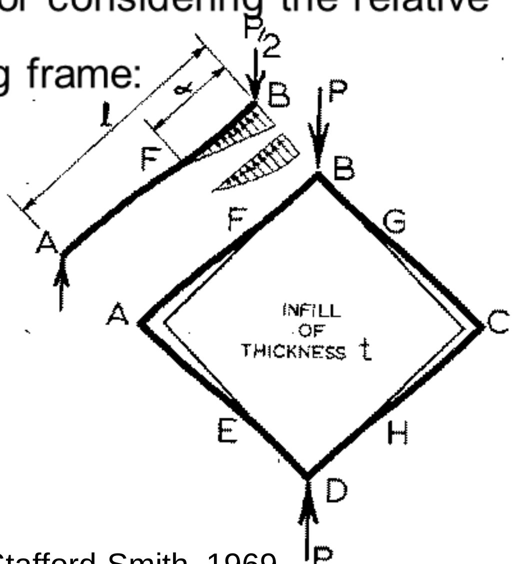
LOCAL EFFECTS PRODUCED BY INFILL WALLS ON THE RC FRAME

- The evaluation of the equivalent width, *w*, varies from one reference to the other
- Paulay and Priestley (1992) → w = 0.25 d_m
- Stafford-Smith (1969) proposed a dimensionless parameter ***λl*** for considering the relative flexural stiffness of the infill to that of the columns of the confining frame:

$$\lambda l = l \frac{\sqrt[4]{E_m t}}{4 E_c I_{col} l'}$$

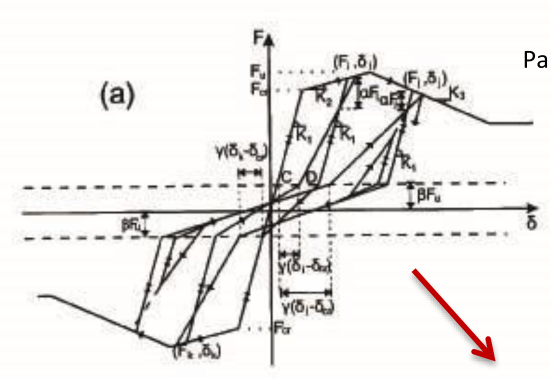
- this parameter is then used for calculating the contact length ***α***:

$$\frac{\alpha}{l} = \frac{\pi}{2\lambda l}$$

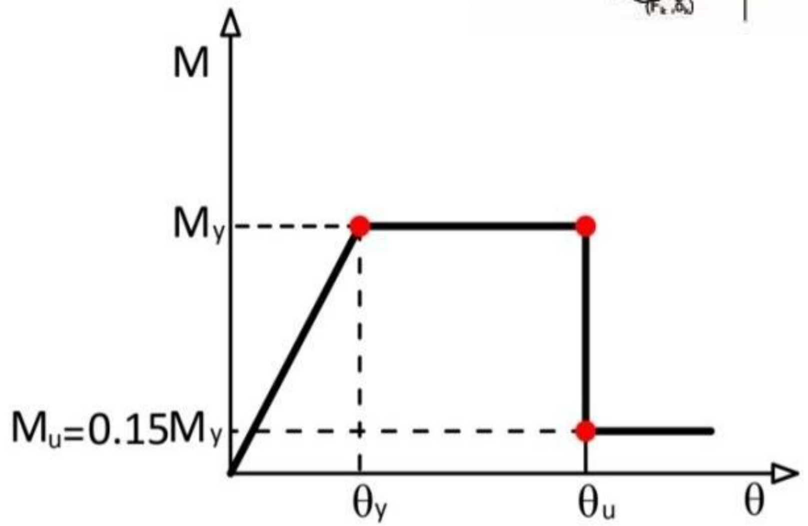


Stafford-Smith, 1969

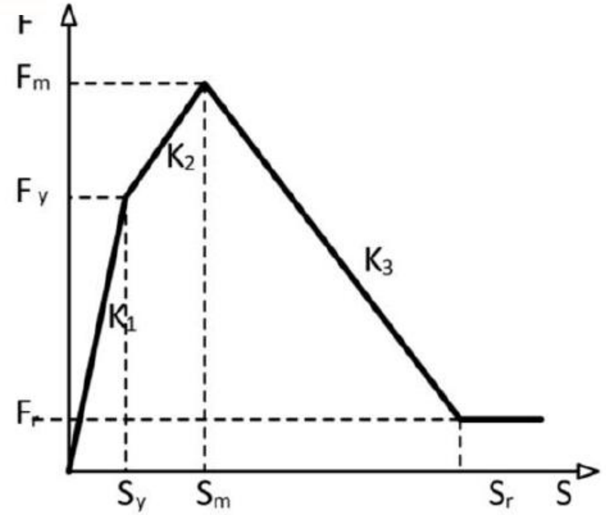
PUSHOVER ANALYSIS



Panagiotakos & Fardis, 1996



RC Beam & column: Moment-rotation law



Infill walls: Displacement - base shear law

Non-Linear Static (pushover) Analysis

The method is based on the application of incremental horizontal force systems to the structure under exam, simulating the effects of inertial seismic forces.

At least two vertical distributions of the lateral loads should be applied (§7.3.7.2 N.T.C.):

1) Modal pattern proportional to lateral forces consistent with the lateral force distribution in the direction under consideration determined in elastic analysis

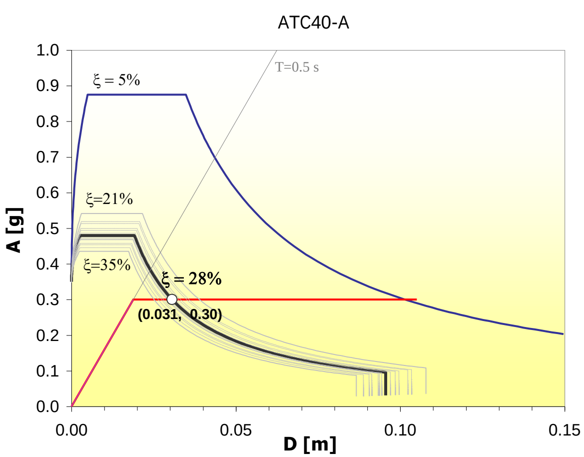
2) Uniform pattern based on lateral forces that are proportional to mass regardless of elevation (uniform response acceleration)

Main difference between the various NLSA proposed methods is related to the use of design spectrum

ATC 40, FEMA 273

Elastic overdamped spectrum

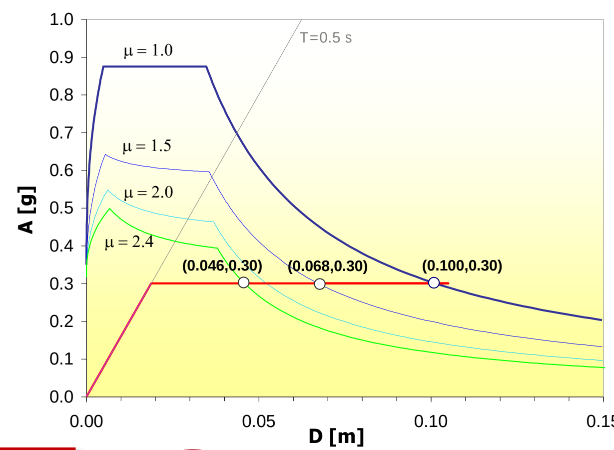
$$\hat{\varsigma}_{eq} = \varsigma + \varsigma_{eq}$$



EC8

Inelastic spectrum

$$R_{\mu} = \frac{mS_{Ael}(T,\xi)}{F_y(\mu = \mu_i)}$$



Procedure comprises the following steps:

1. Create structural model with non-linear elements

   Concentrated or distributed plasticity

2. Incremental analysis for pushover curve

   – incremental horizontal forces systems $\boxed{\mathbf{P} = p\mathbf{M\Phi}}$

   – Displacement D_t of control point (center of mass of last level for buildings)

3. Convert M-DOF system in a S-DOF system and obtain capacity curve

   modal participation factor Γ is used

   in order to scale forces and displacements $\boxed{\Gamma = \frac{\mathbf{\Phi}^T \mathbf{Ms}}{\mathbf{\Phi}^T \mathbf{M\Phi}}}$

   $D^* = \frac{Dt}{\Gamma} \qquad F^* = \frac{V}{\Gamma}$

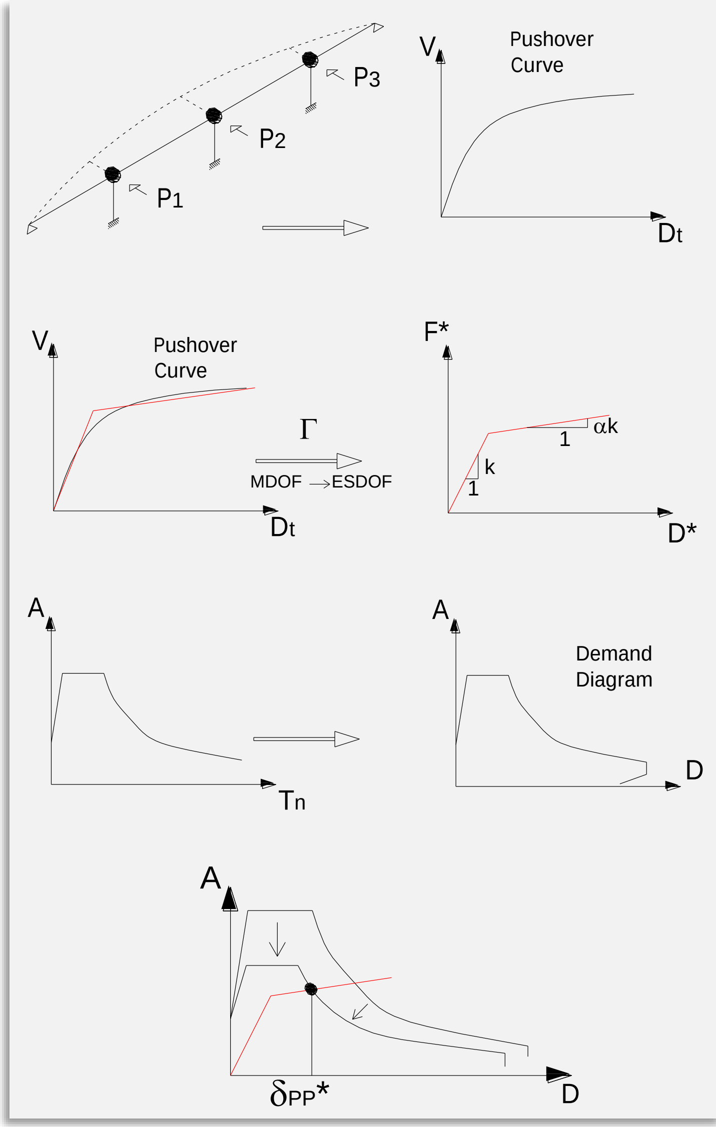
4. Get seismic ground motion demand in the Acceleration-Displacement Response Spectrum (ADRS) format

   Spectrum scaling factor $D = \frac{T_n^2}{4\pi^2} A$

5. Determination of performance point

   Intersection between capacity curve and demand curve in ADRS space

6. Compute displacement for M-DOF system



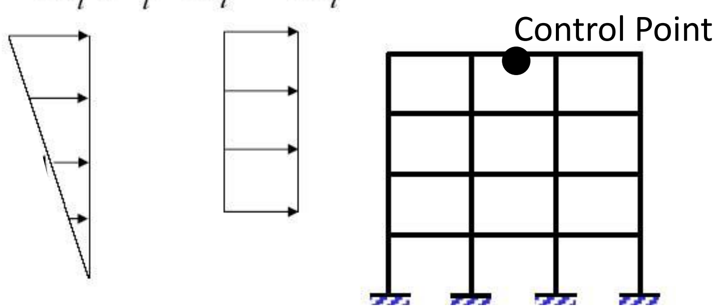
Non-Linear Static (pushover) Analysis

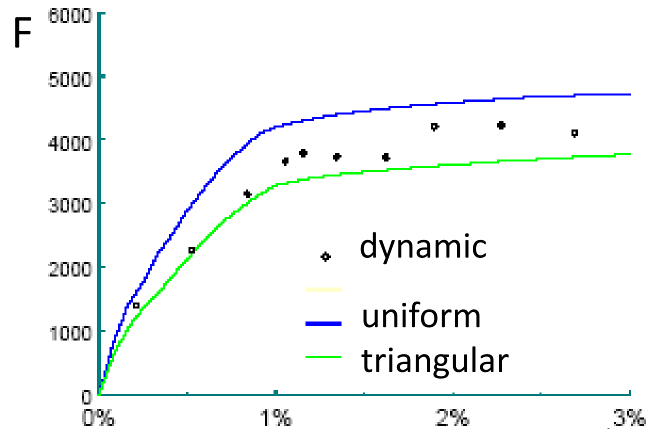
SHORTCOMINGS

LOAD PATTERN ⟶ simulate inertial forces

- the use of load pattern based on the fundamental mode shape may be inaccurate if higher modes are significant
- the use of a fixed load pattern may be unrealistic if yielding is not uniformly distributed, so that the stiffness profile changes as the structure yields

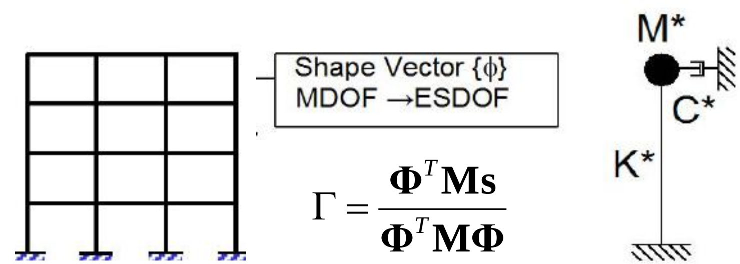
$F_i = m_i \Phi^1_i \qquad F_i = m_i$





REPRESENTATIVENESS OF THE ESDOF

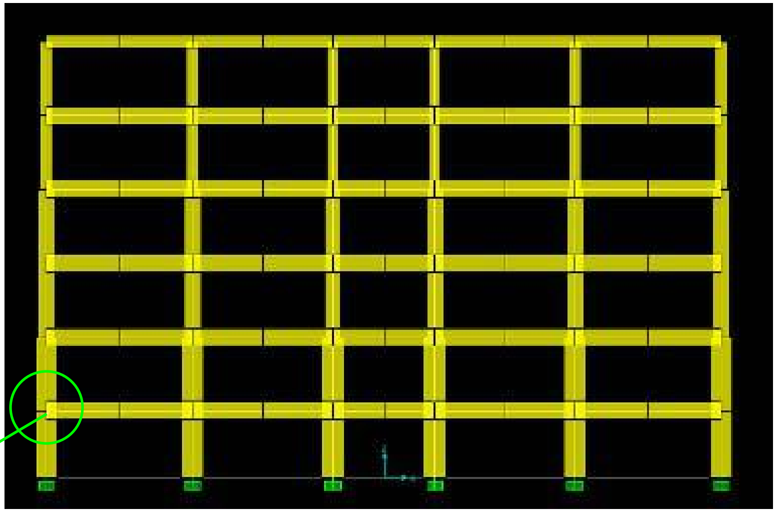
- the seismic response of the original MSDOF system cannot be adequately represented by a simple equivalent SDOF in the case of irregular structures, whose dynamic behaviour is affected by multiple modes of vibration



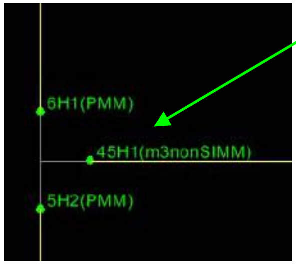
Non-Linear Static (pushover) Analysis

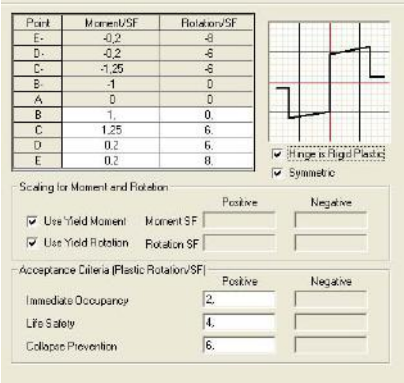
The model used for the case study of the multi-storey RC frame is a concentrated-plasticity model and was created with Sap2000: the software automatically creates 5 points moment-rotation curves based on reinforcement bars in the sections. The points are: origin (A), yielding (B), failure (C), residual strength (D), ultimate (E).

2D model



Flexural plastic hinges M3 have been defined for the beams; columns have been provided with P-M2-M3 plastic hinges, which consider the interaction between flexure and axial force.





Point	Moment/SF	Rotation/SF
E-	-0,2	-8
D-	-0,2	-6
C-	-1,25	-6
B-	-1	0
A	0	0
B	1,	0,
C	1,25	6,
D	0,2	6,
E	0,2	8,

Hinge is Rigid Plastic
Symmetric

Scaling for Moment and Rotation

	Positive	Negative
Use Yield Moment — Moment SF		
Use Yield Rotation — Rotation SF		

Acceptance Criteria (Plastic Rotation/SF)

	Positive	Negative
Immediate Occupancy	2.	
Life Safety	4.	
Collapse Prevention	6.	

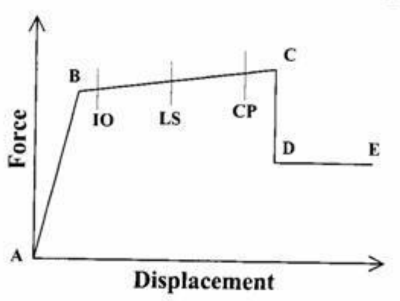
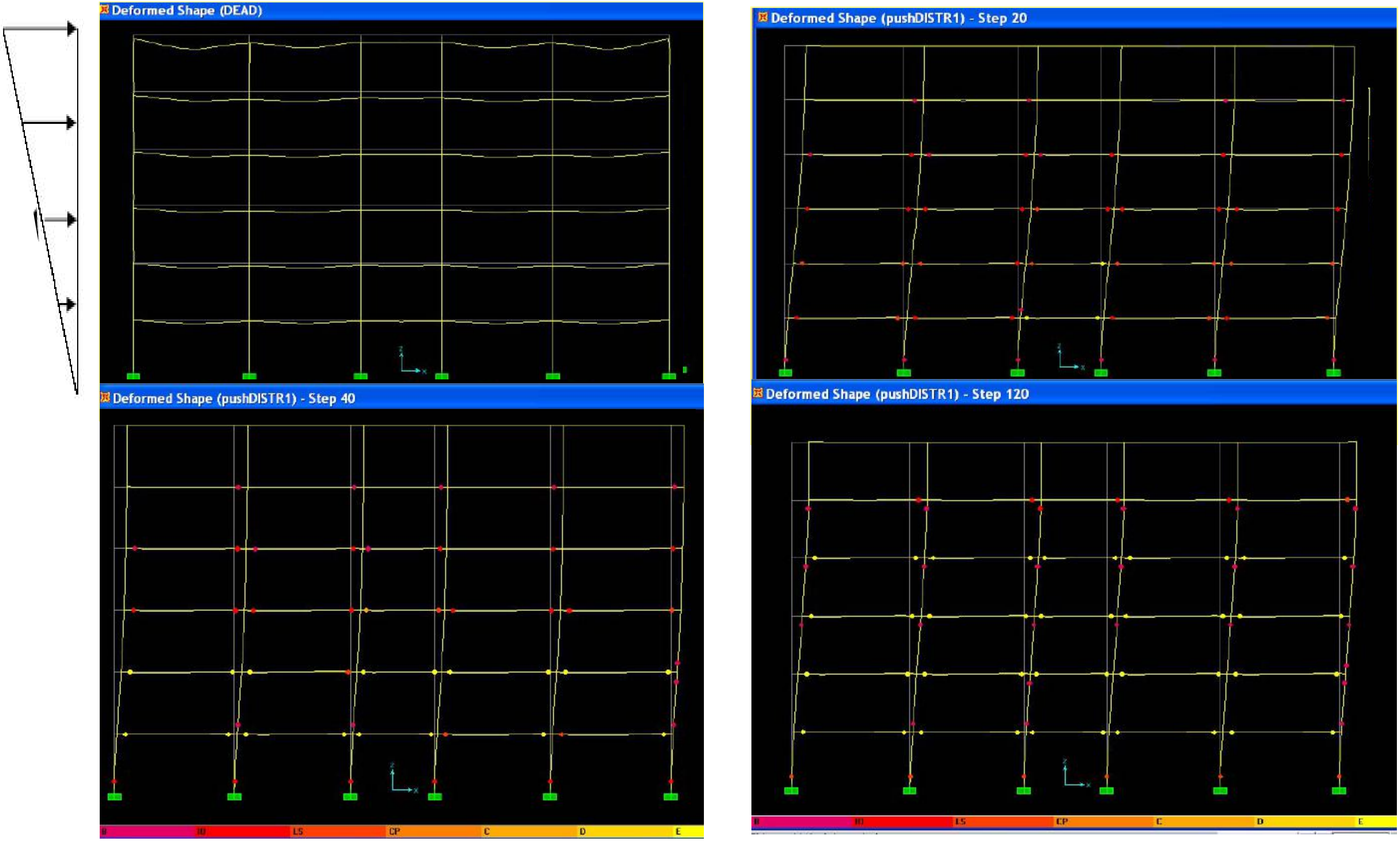


Figure 30
The A-B-C-D-E curve for Force vs. Displacement
The same type of curve is used for Moment vs. Rotation

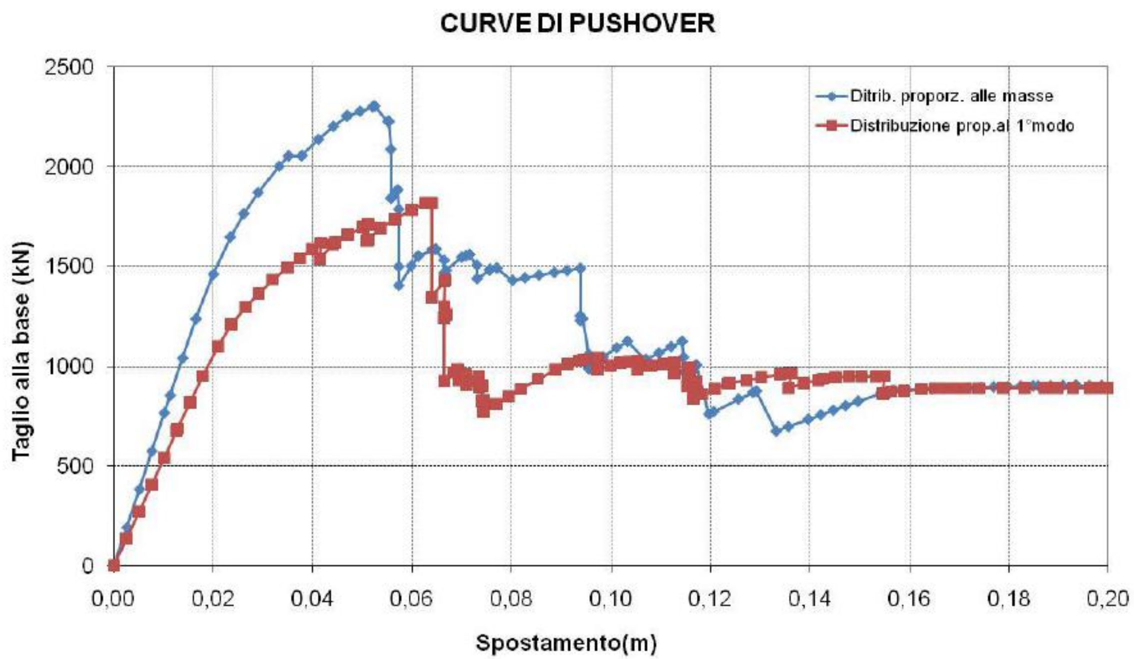
Moment distribution near collapse

Non-Linear Static (pushover) Analysis

Development of plastic hinges while horizontal forces increase during the analysis



Non-Linear Static (pushover) Analysis



F_{bu}*	1309,32	(kN)
0,6 F_{bu}*=	785,59	(kN)
E_m*=	42,46	(kNm)
K_y=	52466,12	(kN/m)
d_y*(m)=	0,022	
$m^*(t)=\sum m_i\phi_i$=	1152,54	

$$T^* = 2 * \pi \sqrt{\frac{m^*}{k^*}} = \mathbf{0{,}93} \quad \text{(s)}$$

F_{bu}* *resistenza max sistema SDOF equiv.*

0,6 F_{bu}*= *ordinata punto di passaggio curva bilineare*

E_m*= *area sottesa dalla curva di capacità (sistema SDOF equiv.) fino allo spostamento du **

K_y= *rigidezza secante*

d_y*(m)= *spostamento allo snervamento*

Force distribution proportional to first mode of vibration

$$\phi_1 = \begin{bmatrix} 1{,}00 \\ 0{,}86 \\ 0{,}65 \\ 0{,}47 \\ 0{,}26 \\ 0{,}09 \end{bmatrix} \qquad M = \begin{bmatrix} 287{,}80 & & & & & \text{Ø} \\ & 363{,}83 & & & & \\ & & 369{,}33 & & & \\ & & & 374{,}84 & & \\ & \text{Ø} & & & 380{,}34 & \\ & & & & & 378{,}47 \end{bmatrix} \qquad R = \begin{bmatrix} 1{,}00 \\ 1{,}00 \\ 1{,}00 \\ 1{,}00 \\ 1{,}00 \\ 1{,}00 \end{bmatrix}$$

$M_1^* = \phi_1^T M \phi_1 =$ 829,68 (t)

$\gamma_1 = (\phi_1^T MR)/(M_1^*) =$ **1,39**

- Determination of the displacement demand for the inelastic system and conversion of the displacement of ESDOF system in the real deformed shape of the structure.
- In case $T^* > T_c$ the displacement response of the inelastic system (GDL-1) is equal to that of an elastic system of equal period and is obtained with the expression:

$T^* \geq T_c$

$d^*_{max} = S_{Da}(T^*) = S_{De}(T^*)$

$$S_{De}(T^*) = S_{Ae}(T^*)\left(\frac{T^*}{2\pi}\right)^2 = a_g * S * \eta * 2.5\left(\frac{T_c}{T^*}\right)\left(\frac{T^*}{2\pi}\right)^2 = d^*_{max} = \quad 0{,}127$$

The displacement of the M-DOF system at the control point is obtained as: $D_c = D^*\Gamma =$ $d_{max}(m) =$ 0,176

Verifications of the structural elements in terms of ductility and displacement capacity (§7.3.6.2 N.T.C.)

- It can be observed that with increasing $F_{horizontal}$ plastic hinges form at beams ends and there aren't cases of anticipated failures in the columns, in accordance with capacity design principles.
- It can also be observed that the behaviour factor assumed in the modal response spectrum analysis is similar to the one obtained with non-linear static analysis.

Non-Linear Time History Analysis (NLTHA)

Equation of motion is directly solved with numerical integration:

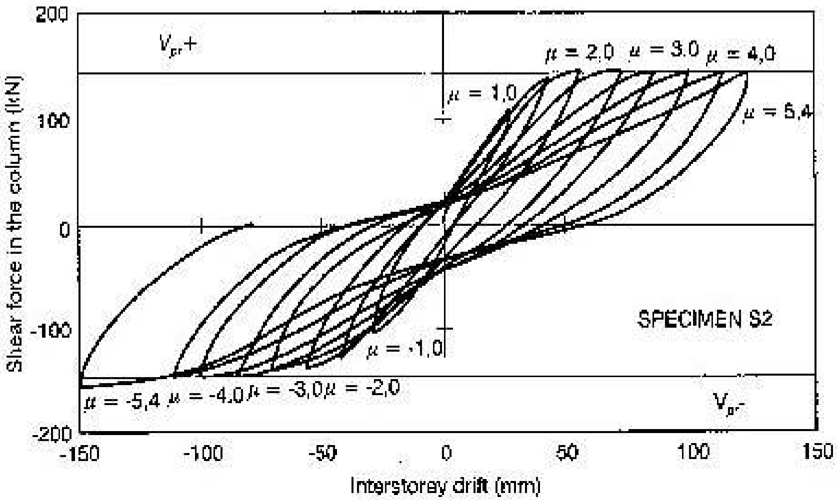
$$\mathbf{M} \cdot \ddot{\mathbf{U}}(\mathrm{t})+\mathbf{C} \cdot \dot{\mathbf{U}}(\mathrm{t})+\mathbf{K_s}(\mathbf{U},\mathrm{t},\mathrm{T}) \cdot \mathbf{U}(\mathrm{t})=\mathbf{F}(\mathrm{t})$$

- Differently from analysis of linear system secant stiffness matrix $\mathbf{K_s}$ (and thus internal force vector $\mathbf{R}$) is variable over time depends on
  - displacement vector $\mathbf{U}$ (unknown)
- Mass matrix $\mathbf{M}$ and damping matrix $\mathbf{C}$ could also be variable!
- The numerical problem is non linear and requires iterative methods (explicit or implicit scheme for the solution (e.g. Newmark's method, unconditionally stable)

$$\mathbf{R}(\mathbf{U},\mathrm{t},\mathrm{T}) = \mathbf{K_s}(\mathbf{U},\mathrm{t}) \cdot \mathbf{U}(\mathrm{t})$$

$\mathbf{K_s}$ secant stiffness matrix

$\mathbf{R}$ internal force vector



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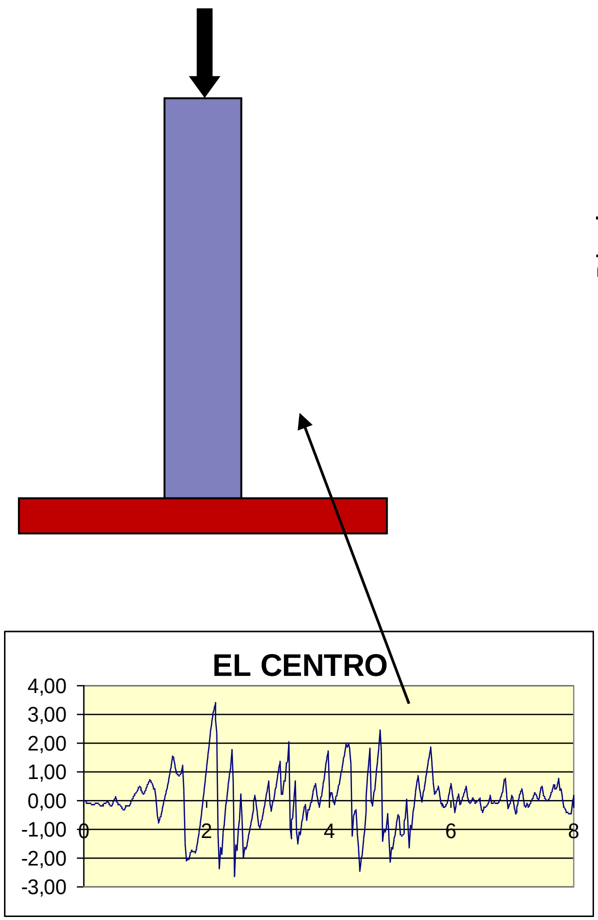
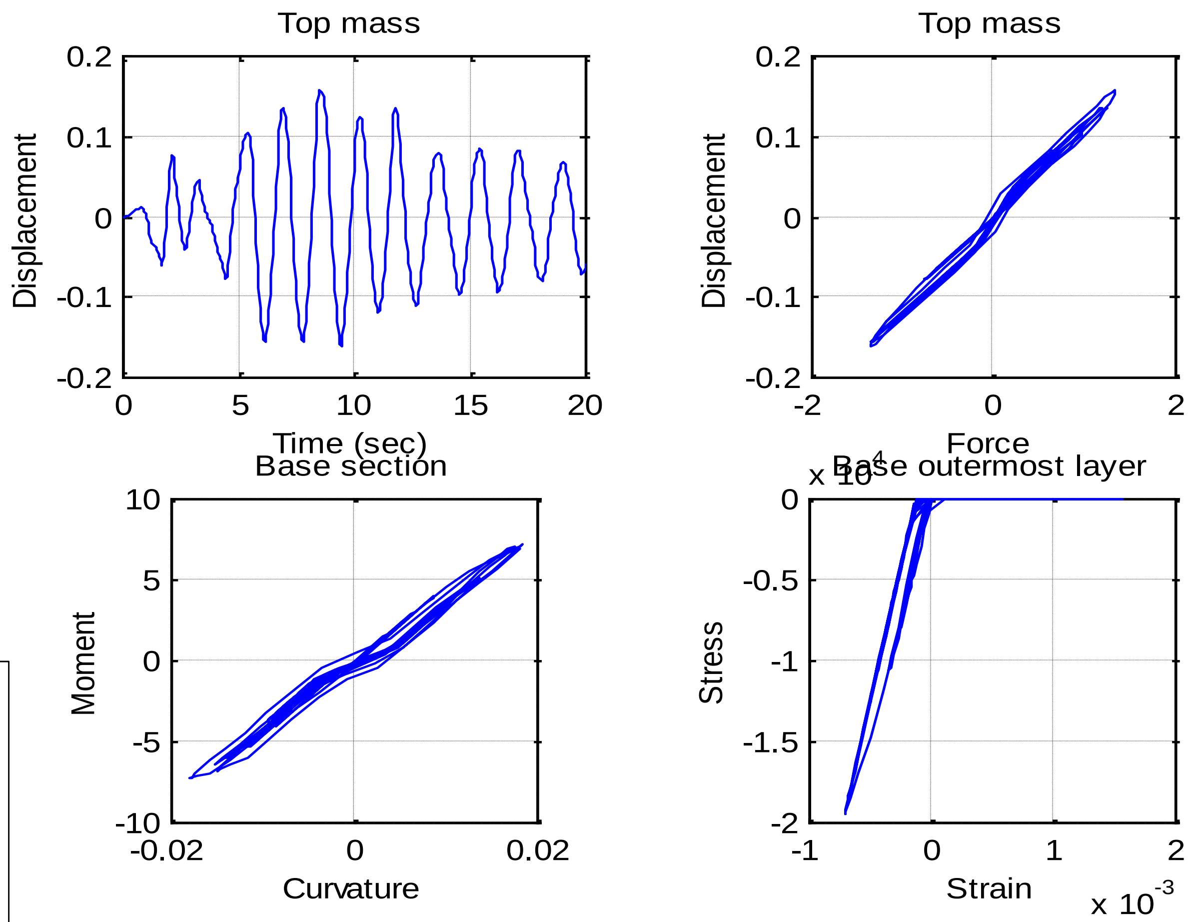


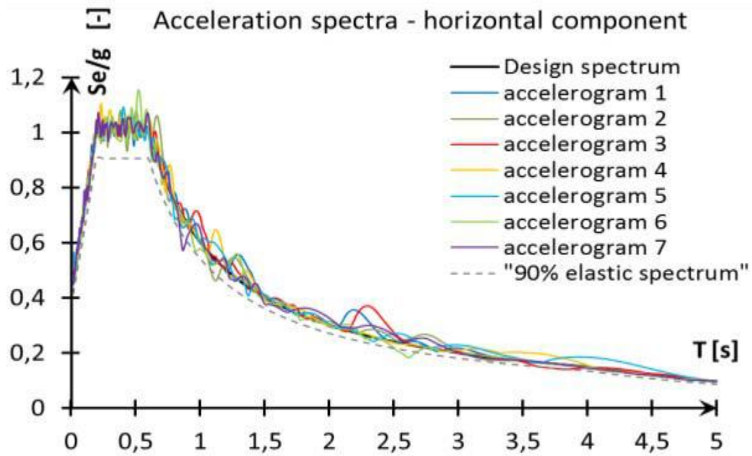
EL CENTRO
4,00
3,00
2,00
1,00
0,00
-1,00
-2,00
-3,00
0
2
4
6
8



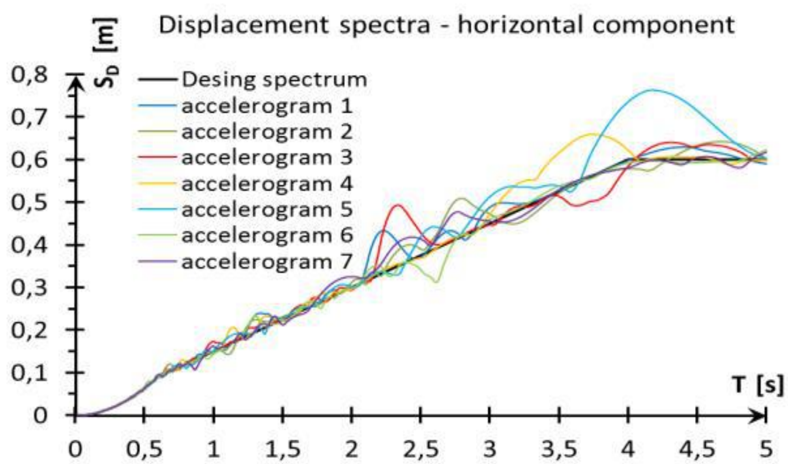
Top mass
Displacement
Time (sec)
0.2
0.1
0
-0.1
-0.2
0
5
10
15
20
Top mass
Displacement
Force
-2
0
2
x 10^4
Base section
Moment
Curvature
10
5
0
-5
-10
-0.02
0
0.02
Base outermost layer
Stress
Strain
0
-0.5
-1
-1.5
-2
-1
0
1
2
x 10^-3

Non-Linear Time History Analysis (NLTHA)

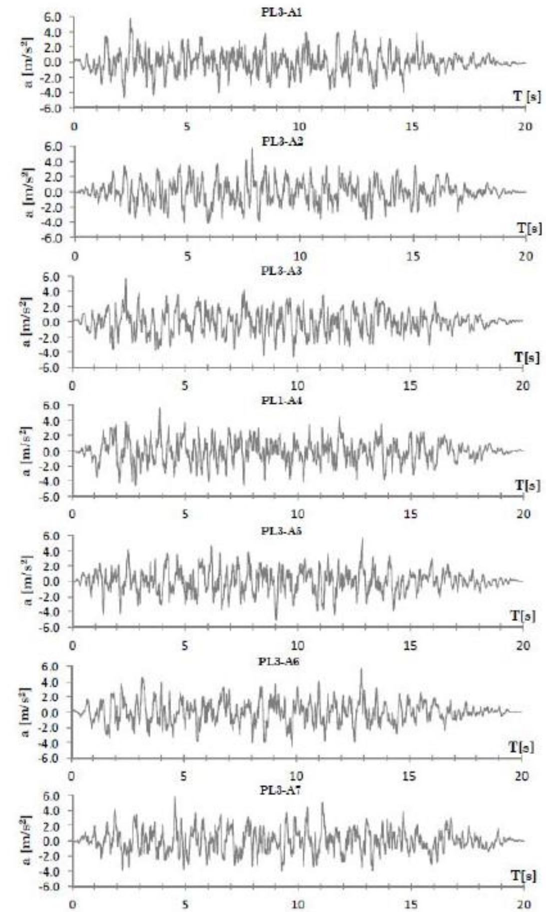
Can be derived as SPECTRUM-COMPATIBLE GROUND MOTIONS from the smoothed **elastic spectrum of EC 8**



ARS



DRS



type C soil (S=1.15, TB=0.20s TC=0.6s, TD=2.0s)

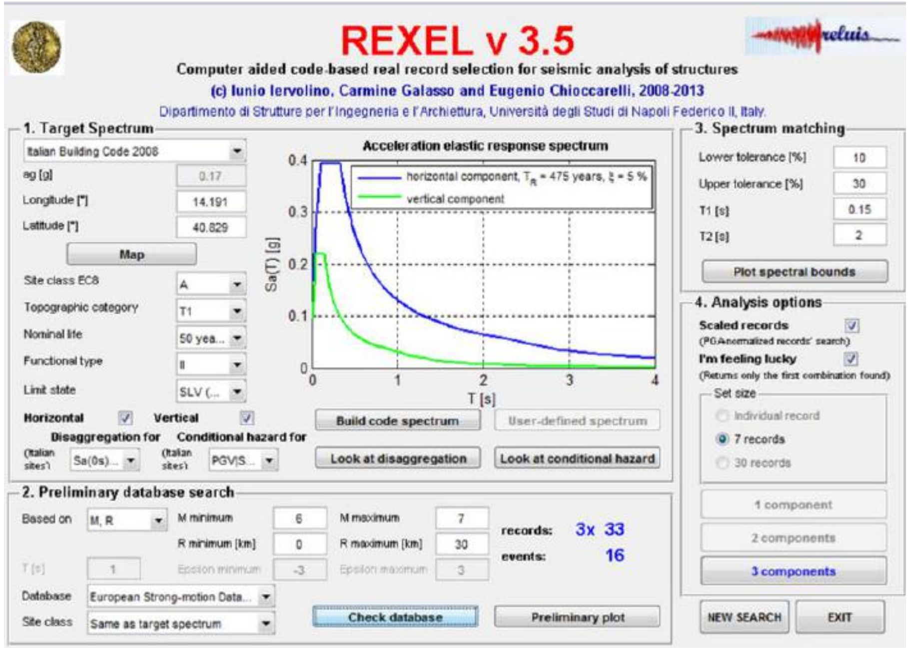
Strong Ground Motion Database

As alternative sets of natural Recorded Ground Motions can be adopted. There are several databases on line:

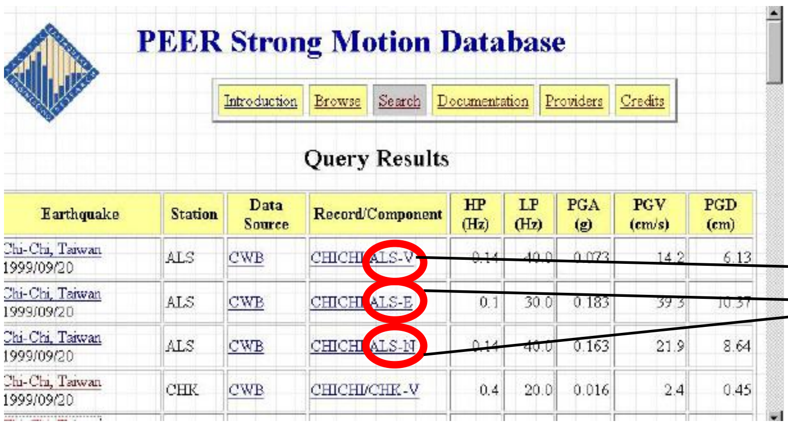
PEER Strong Ground Motion Database

- The Pacific Earthquake Engineering Research (PEER), headquartered at the **University of California at Berkeley**, makes available online over 10,000 strong ground motion records from 173 different earthquakes

http://peer.berkeley.edu/



If you use REXEL, please cite it as: Iervolino I., Galasso C., Cosenza E. (2009). REXEL: computer aided record selection for code-based seismic structural analysis. Bulletin of Earthquake Engineering, 8:339-362. DOI 10.1007/s10518-009-9146-1



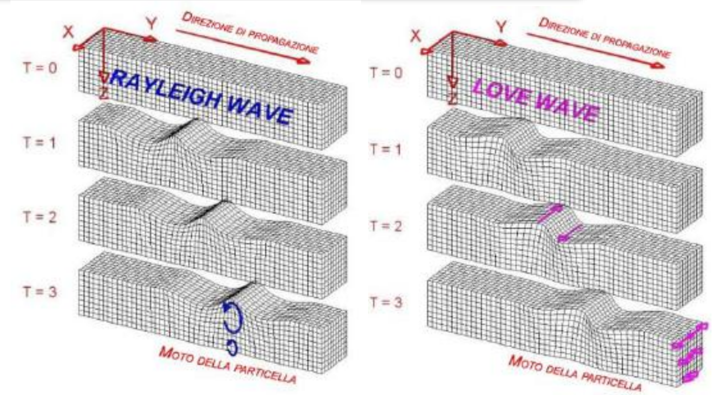
Earthquake	Station	Data Source	Record/Component	HP (Hz)	LP (Hz)	PGA (g)	PGV (cm/s)	PGD (cm)
Chi-Chi, Taiwan 1999/09/20	ALS	CWB	CHICHI/ALS-V	0.14	40.0	0.073	14.2	6.13
Chi-Chi, Taiwan 1999/09/20	ALS	CWB	CHICHI/ALS-E	0.1	30.0	0.183	39.3	10.37
Chi-Chi, Taiwan 1999/09/20	ALS	CWB	CHICHI/ALS-N	0.14	40.0	0.163	21.9	8.64
Chi-Chi, Taiwan 1999/09/20	CHK	CWB	CHICHI/CHK-V	0.4	20.0	0.016	2.4	0.45

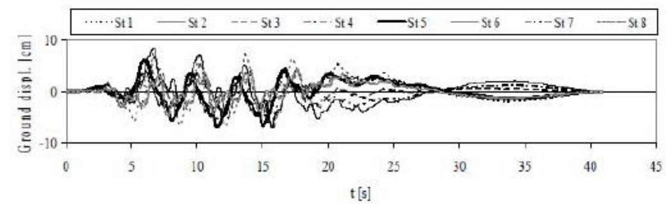
Vertical components

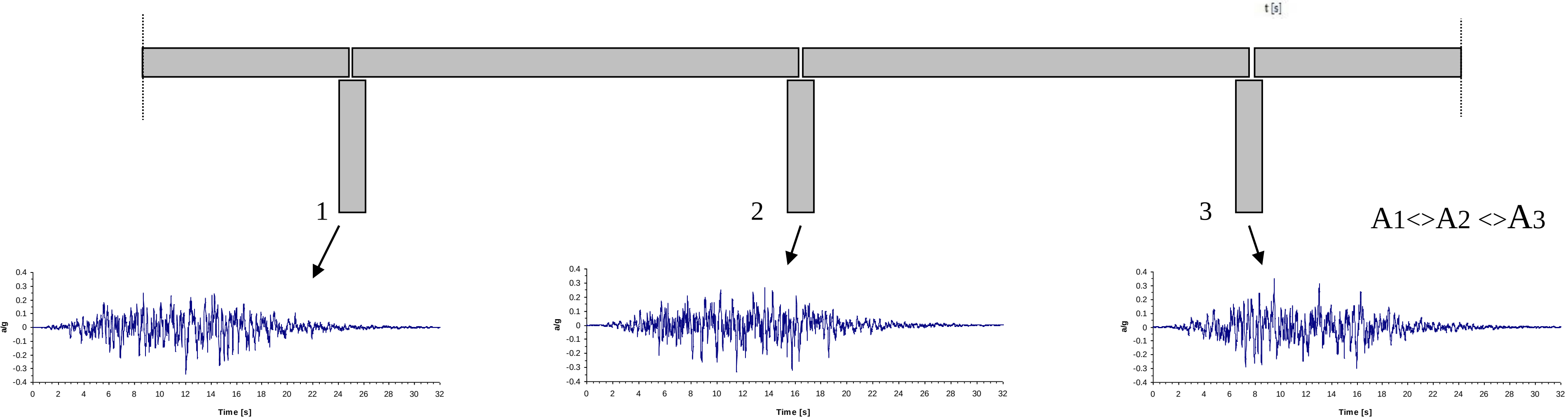
Horizontal components

Non-Linear Time History Analysis (NLTHA)

For spatial long-extending structures additional effects should also be considered, related to spatial variability of earthquakes: spatial variation properties of the ground motions (loss of coherence, time delay etc.) can significantly change the structural response especially in terms of pounding forces and possible increment of relative displacements (unseating)

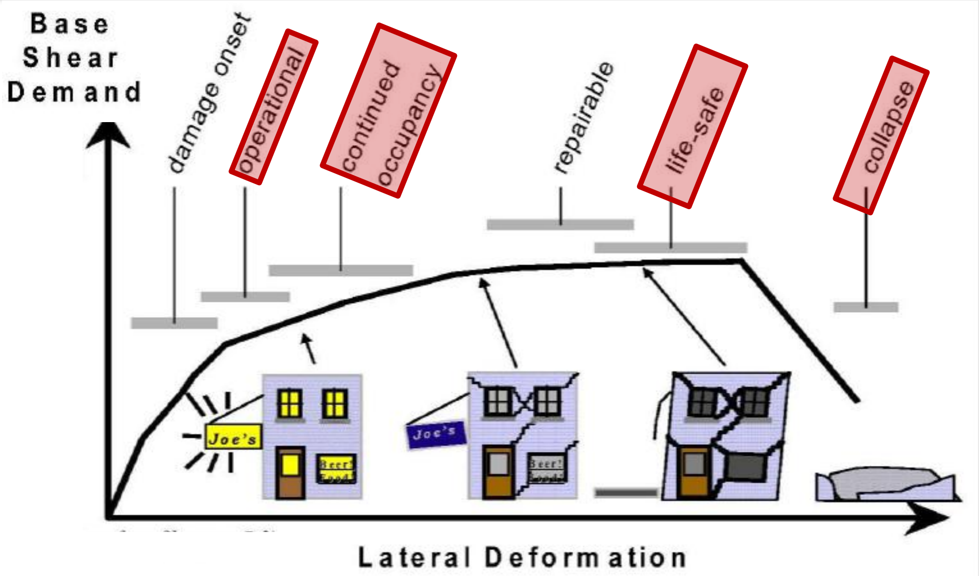






- 3. DISPLACEMENT-BASED METHODS

Introduction to DBD



PERFORMANCE CRITERIA:

- **FORCE-BASED METHODS (FBD)**: **SHEAR DEMAND**

- **DISPLACEMENT-BASED METHODS**(DBD): **DESIGN DISPLACEMENT** (strain or drift limits)

Performance should be directly related to displacement measures:

Δ– global,

δ–interstory relative displacement

$\Theta=\delta/h$ Interstory drift (rotation)

Displacement is the fundamental index of structural damage

7. DISPLACEMENT-BASED SEISMIC DESIGN

Damage-control limit state - displacements rather than forces are used as measurement of earthquake damage

Two methods can be employed:

- The traditional force based design approach (starts by proportioning the structure for strength and stiffness) combined with required displacement target verification;

- The direct displacement based design approach in which the design starts from the target displacements. Then the analysis is performed and determined strength and stiffness (as the end result of the design process) to achieve the design displacements.

Introduction to DBD

PERFORMANCE CRITERIA:

Level 1 ("Serviceability")

Level 2 ("Damage Control")

Level 3 ("Collapse Prevention")

ε_m STRAIN LIMITS

Material	Level 1	Level 2	Level 3
Concrete comp. strain	0.004	Eq.(2.2) <0.02	1.5Eq.(2.2)
Re-bar tension strain	0.015	$0.06\varepsilon_{su}$ <0.05	$0.09\varepsilon_{su}$ <0.08
Structural Steel Strain - Class 1 Sections, Flexural Plastic Hinges	0.010	0.025	0.04
Structural Steel strain - Class 2* & 3 Sections, Flexural Plastic Hinges	ε_y	ε_y	ε_y
Steel Brace Deformation Limits (see Eq. 2.3)	$\chi_{br}\varepsilon_y$	$0.25\mu_f\varepsilon_y$	$0.5\mu_f\varepsilon_y$
Reinforced Masonry comp. strain	0.003	Eq.(2.2) <0.01	1.5Eq.(2.2)
Uneinforced Masonry** comp. strain	0.003	0.004	0.004
Timber tension strain	$0.75\varepsilon_y$	$0.75\varepsilon_y$	$0.75\varepsilon_y$
Structural Elements of Isolated Structures	As per Section 2.4.3		

*higher strain limits may be used for Class 2 sections if the values are supported by adequate experimental data.
**values may differ depending on type of masonry – check with manufacturer wherever possible.

θ_c DRIFT LIMITS

Drift Limit	Level 1	Level 2	Level 3
Buildings with brittle non-structural elements*	0.004+	0.025	No limit
Buildings with ductile non-structural elements*	0.007+	0.025	No limit
Buildings with non-structural elements detailed to sustain building displacements*	0.010+	0.025	No limit
Framed timber walls	0.010	0.020	0.030
RC Bridge Piers**	θ_y	0.03	0.04
Isolated Bridges	$2/3{*}\theta_y$	$2/3{*}\theta_y$	θ_y

+ *For the design of base isolated buildings, the performance Level 1 drift limits for fixed-base buildings shall be used for all design intensity levels.*
* *Shear design of flat slab systems should account for design drifts shown, or lower drift limits adopted.*
***Optional – see commentary. θ_y = pier yield drift.*

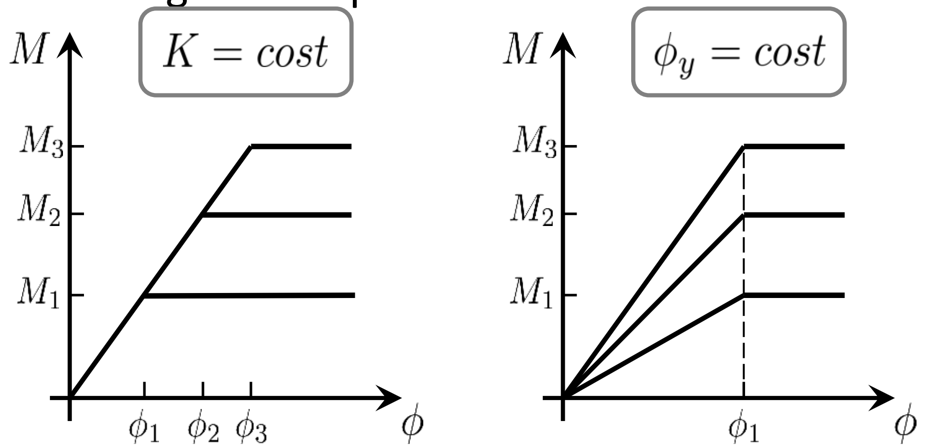
Criticis to FBD -summary

1. **INTERDEPENDENCY OF STRENGHT AND STIFFNESS**:

- The yield point remains almost the same as the strength increases (the yield curvature is the independent parameter)
- Strength and stiffness are closely related each other (member strength influences member stiffness)

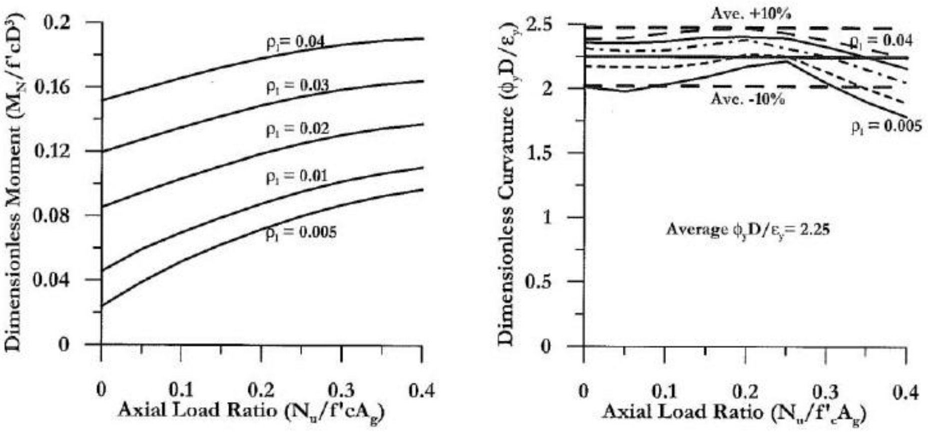
CONSEQUENCES

- Estimation of elastic structural period is questionable
- Distribution of required strength through the structure is doubtful
- Member strength demand is the end product of FBD →Iterative process



Example: Circular column $\phi_y = 2.25\,\varepsilon_y/D$

Rectangular column $\phi_y = 2.10\,\varepsilon_y/h_c$



(a) Nominal Moment

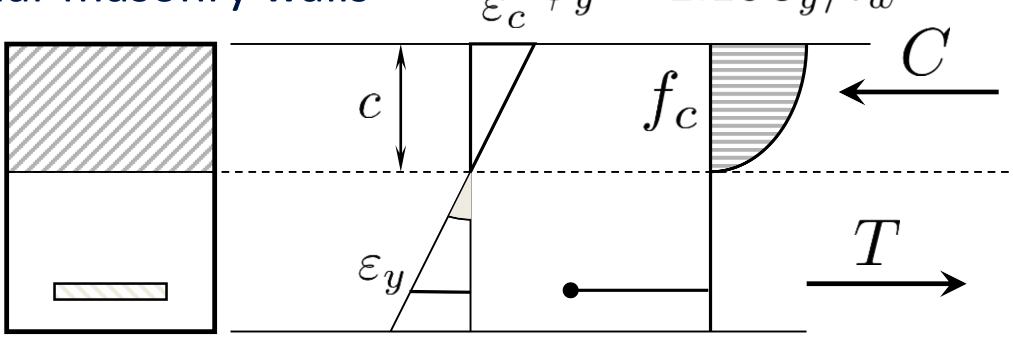
(b) Yield Curvature

Ingredients for DDBD

YIELD DEFORMATIONS:

- Stiffness and strength are effectively proportional for a give structural member. The independent parameter for calculations is thus the yield displacement, or alternatively the yields curvature.

Circular column	$\phi_y = 2.25\,\varepsilon_y/D$
Rectangular column	$\phi_y = 2.10\,\varepsilon_y/h_c$
rectangular concrete walls	$\phi_y = 2.00\,\varepsilon_y/l_w$
T-Section beams	$\phi_y = 1.70\,\varepsilon_y/h_b$
Flanged concrete walls	$\phi_y = 1.50\,\varepsilon_y/l_w$
Rectangular masonry walls	$\phi_y = 2.10\,\varepsilon_y/l_w$



Criticis to FBD-summary

2. FORCE REDUCTION FACTOR

- In order to take into account the inelastic behaviour of the structure as well as its expected energy dissipation capacity, the spectral elastic ordinates are reduced using a behavior factor q (reduction factor R)
- Relationships between ductility and force reduction factor (i.e. equal displacement, equal energy principle) are not well established
- It is not possible to define a unique reduction factor for a given structural type . Example:

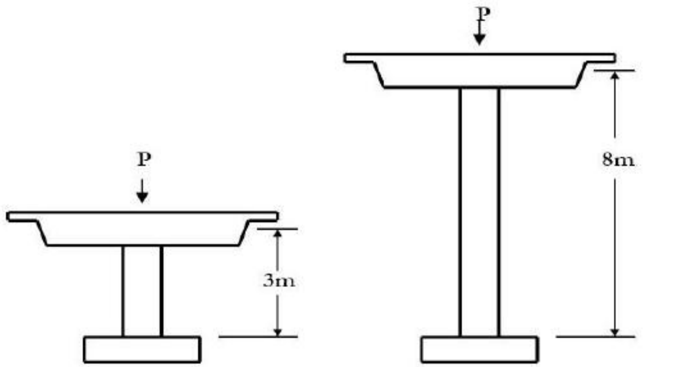
2 bridge piers are considered (same section, same longitudinal reinforcement, different height**)**. Yield curvature ϕ_y and ultimate curvature ϕ_u are equal (depending on section properties).

$\Delta_y = \phi_y H^2/3$ yield displacement , $\Delta_p = \phi_p L_p H$, where:

$\phi_p = \phi_u - \phi_y$ plastic curvature,

L_p = max $(0{,}08H + 0{,}022\ f_y d_{bl};0{,}044\ f_y d_{bl})$

Reduction factor R (FBD method) is the same for both structures, (q=3.5 for D=1m according to DM14.01.08).



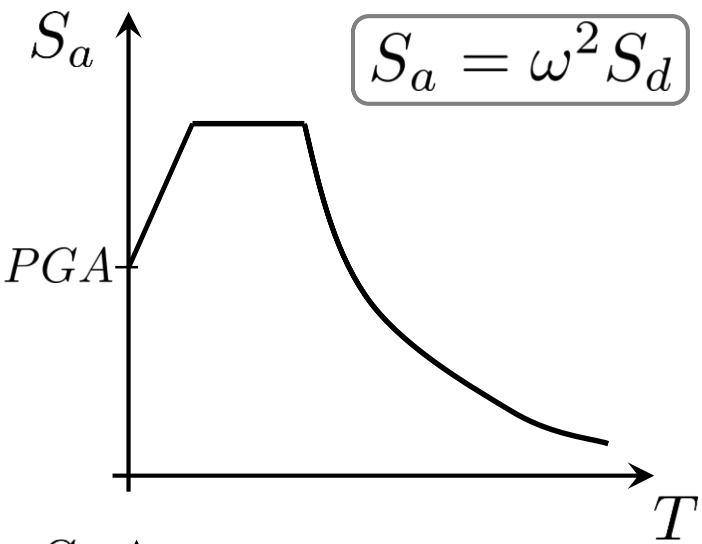
(a) Squat Column, $\mu_\Delta = 9.4$ (b) Slender Column, $\mu_\Delta = 5.1$

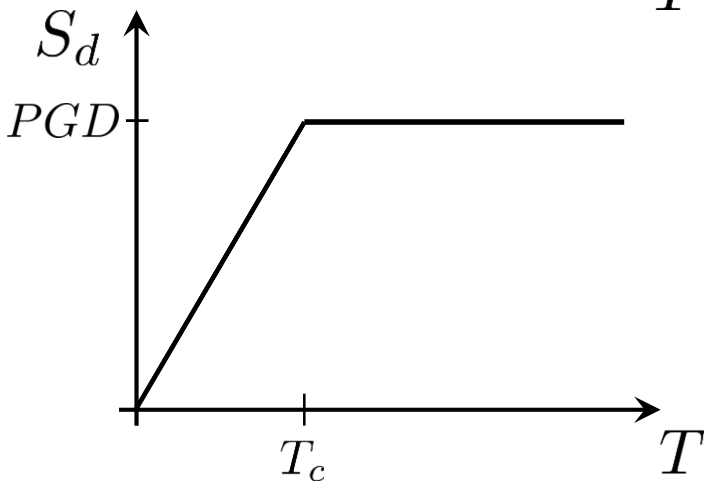
$\mu_\Delta = (\Delta_y + \Delta_p)/\Delta_y = 1 + 3(\phi_p L_p)/(\phi_y H)$ related to the height H of the pier

Ingredients for DDBD

SEISMIC INPUT:

- Elastic displacement spectra (rather then acceleration spectra) are used
- The **displacement response spectrum** describes the maximum elastic response of a set of 1-DOF systems, with continuously varying natural period, to a given ground motion
- The design response spectra are smooth in shape and refer to a bunch of earthquakes. It needs damping reduction factor to account for damping





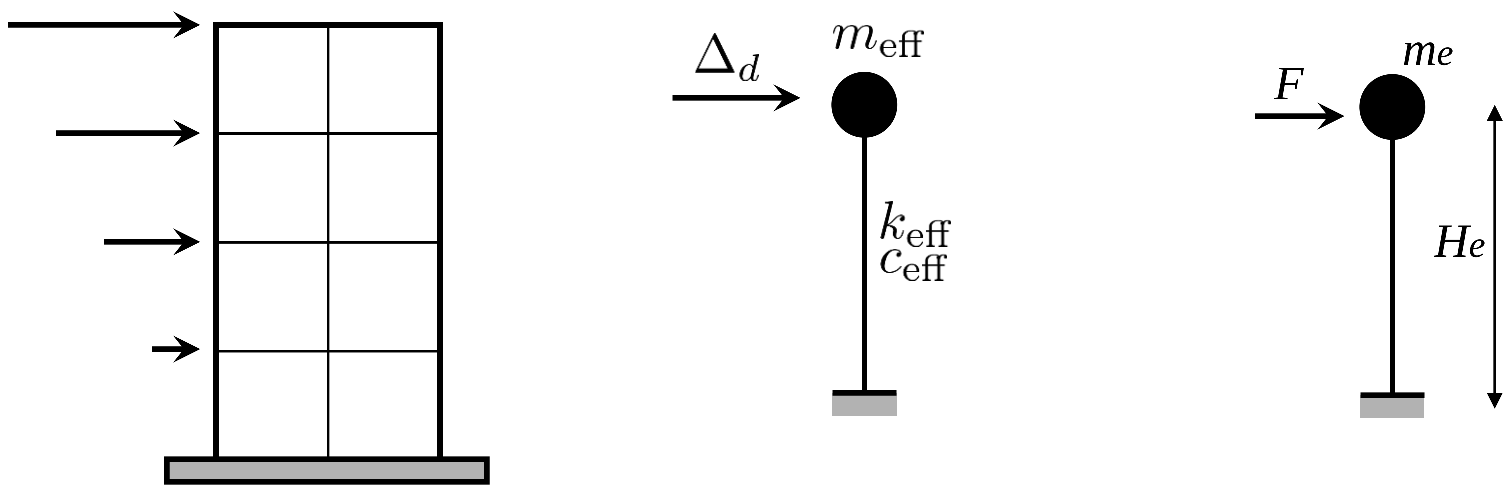
$$S_d(T,\xi) = S_d|_{\xi=5\%}\sqrt{\frac{7}{2+\xi}}$$

Far field source

Ingredients for DDBD

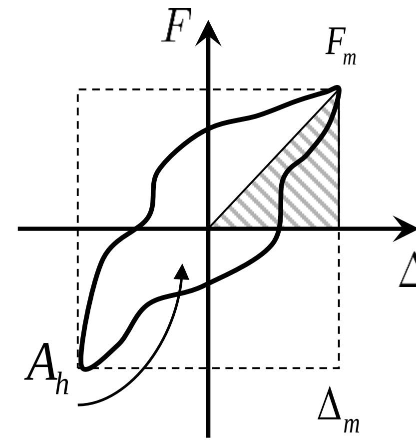
STRUCTURAL MODEL:

- The “substitute structure” is an **equivalent 1-DOF system** characterized by effective stiffness and damping at target displacement



Ingredients for DDBD

- **Equivalent damping:** sum of viscous and hysteretic part according to Jacobsen approach (equates the energy dissipates in the cycle and that dissipated by an equivalent viscous system

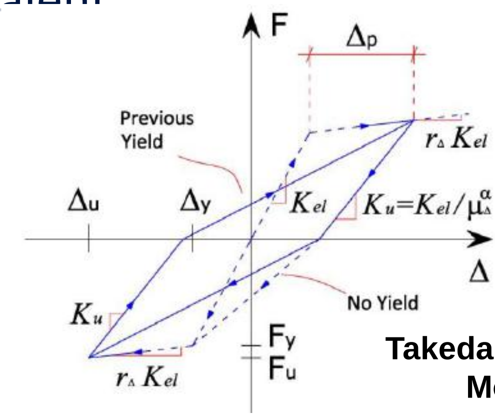


$$\xi_{eq} = \xi_{el} + \xi_{hyst}$$

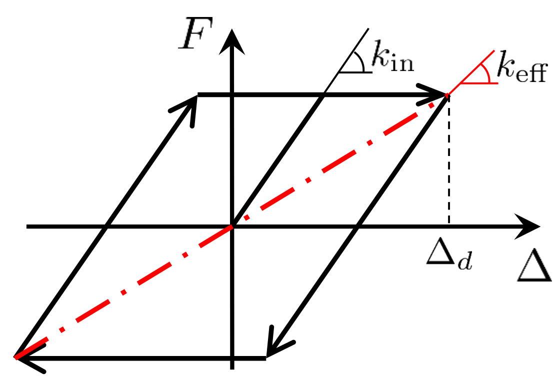
Jacobsen approach

$$\xi_{hyst} = \frac{2}{\pi}\frac{A_h}{2F_m 2\Delta_m} = \frac{2}{\pi}R_a$$

$$\longrightarrow \; \xi_{eq} = 0.05 + 0.444\frac{\mu - 1}{\mu\pi}$$



Takeda Thin (TT) Model

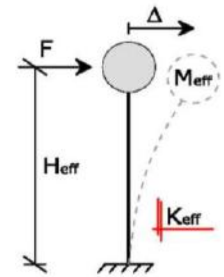


- **Equivalent stiffness:** secant stiffness at the max design displacement (target displacement)

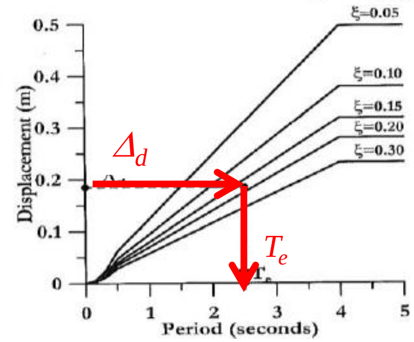
DDBD procedure

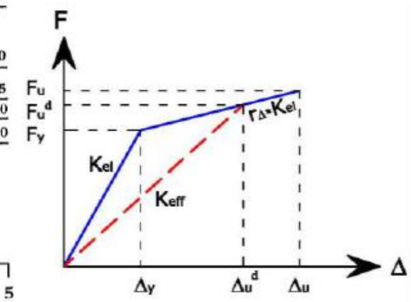
(Priestley et al., 2007)

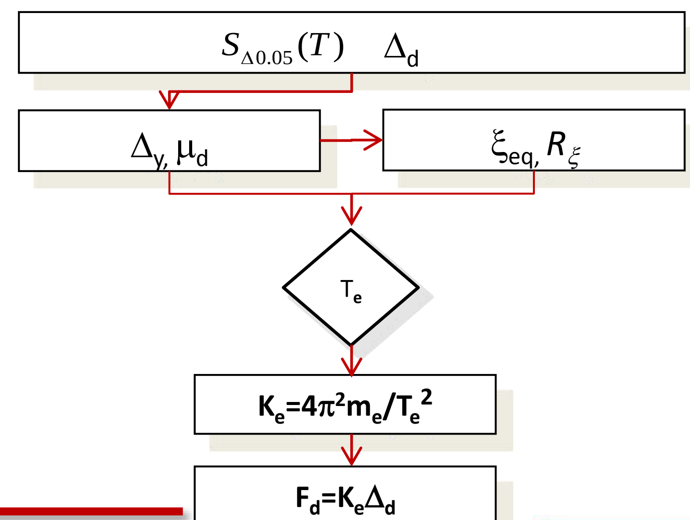
SDOF SYSTEMS



1. Select elastic displ. spectrum and target displacement (strain limit, drift limit, ductility limit) $S_{\Delta 0.05}(T)$ Δ_d
2. Calculate yield displacement $\Delta_y = \frac{\phi_y (H + L_{SP})^2}{3}$
3. Calculate displacement ductility $\mu_\Delta = \Delta_d / \Delta_y$
4. Estimate equivalent damping, resp. spectrum scaling factor $\zeta_{eq} = 0.05 + 0.444\left(\frac{\mu - 1}{\mu\pi}\right)$ $R_\xi = \left(0.10/(0.05+\xi)\right)^{0.5}$
5. Determine T_{eff} from the scaled displacement spectrum $S_{\Delta\xi}(T) = R_\xi S_{\Delta 0.05}(T)$
6. Calculate effective stiffness at max design displacement: $K_{\mathrm{eff}} = 4\pi^2 \frac{m_{\mathrm{eff}}}{T_{\mathrm{eff}}^2}$
6. Calculate base shear and moment
7. Check P-D effects
8. Capacity design





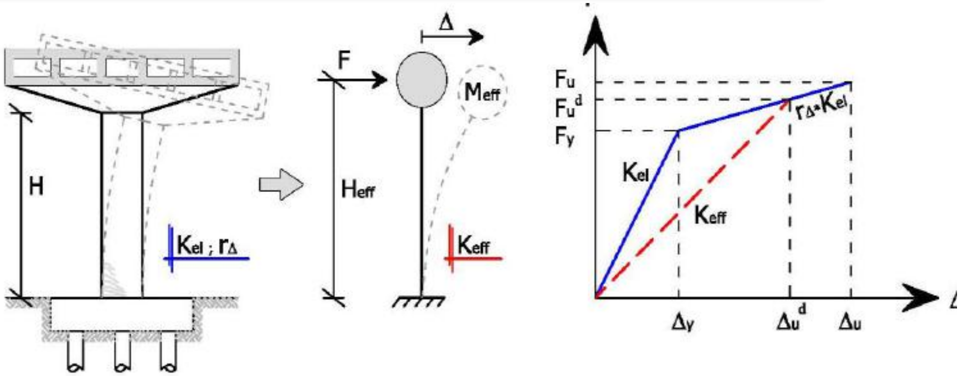


DDBD Example (SDOF system)

An RC column has to be designed for a region of high seismicity, PGA=0.7g.
The effective column height is 10m.

$$M_{eff} = 5000kN,\ D = 2.0m$$

$$Es = 200GPa,\ f_y = 470MPa$$



The design limit state is represented by the more critical of
a) displacement ductility of μ=4
b) drift of θ_D=0.035

DESIGN DISPLACEMENT Δ_d

Yield curvature $\varphi_y = 2.25(470/200000/2.0) = 0.00264\ \dfrac{1}{m}$

Yield displacement $\Delta_y = \dfrac{\phi_y (H + L_{sp})^2}{3} = 0.00264(10^3)/3 = 0.0881m$

(Ignoring for simplicity strain penetration length, L_{sp}=0)

The design displacement is the smaller target displacement value :

a) $\Delta_D = 4(0.0881) = 0.353m$ (almost identical)

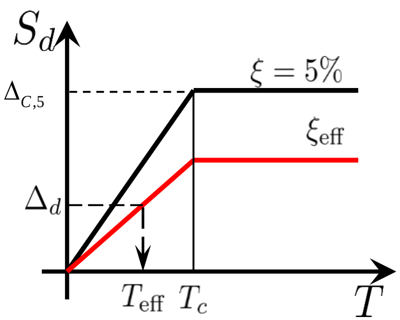
b) $\Delta_D = 0.035(10) = 0.350m$

DDBD Example (SDOF system)

DUCTILITY μ_D AT DESIGN DISPLACEMENT Δ_d $\mu = 0.35/0.0881 = 3.97$

$$\xi_{eq} = 0.05 + 0.0444(3.97-1)/3.97\pi = 0.155\ (15.5\%)$$

MAXIMUM SPECTRAL DISPLACEMENT FOR 5% DAMPING

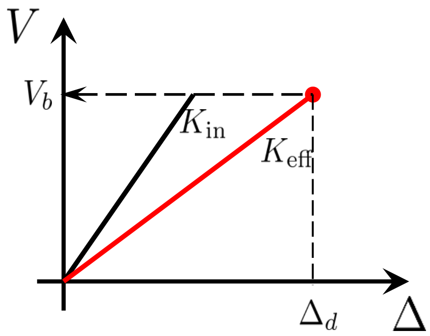


The corner period for a peak displacement response is T_c=4s

Scaling to a PGA of 0.4g, the corresponding displacement is $\Delta_{C,5} = 0.5(0.7)/0.4 = 0.875m$

Applying the damping correction factor, the effective period is obtained by proportion:

$$T_e = T_C \frac{\Delta_D}{\Delta_{c,5}} \sqrt{\frac{0.07}{0.02+0.155}} = 4\frac{0.35}{0.553}\sqrt{\frac{0.07}{0.02+0.155}} = 2.53s$$



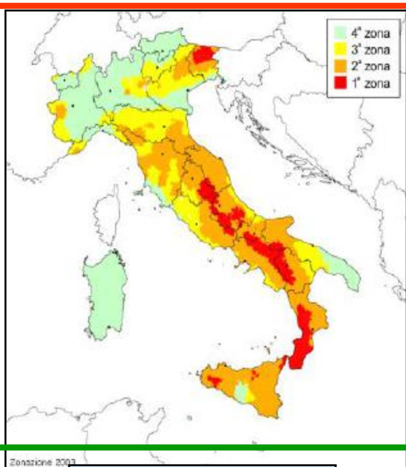
$$K_e = 4\pi^2 m_e / T_e^2 = 4\pi^2 5000/(9.805(2.53)^2) = 3145kN/m$$

DESIGN SHEAR FORCE $V_{base} = K_e \Delta_D = 3145(0.35) = 1100kN$

- 4. PROBABILISTIC APPROACHES FOR SEISMIC RISK ANALYSIS

Probabilistic approach

SEISMIC RISK FACTORS



HAZARD

It's not possible to prevent the earthquakes or to modify their intensity or frequency. The knowledge of the hazard is useful in order to calibrate the interventions. The seismic classification determines the hazard and quantify the reference actions in every area.

VULNERABILITY

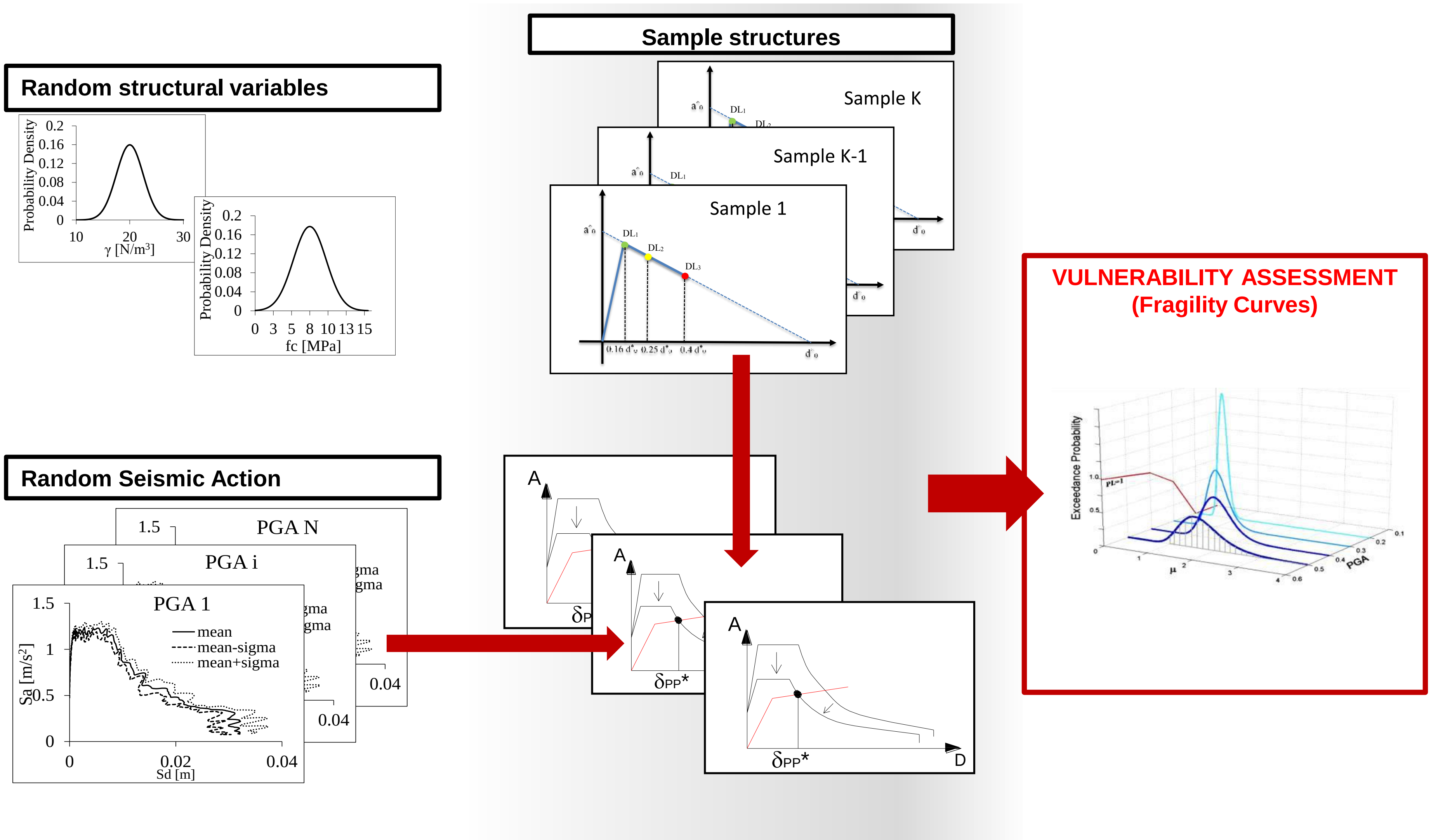
The expected damage is reduced by an improvement of the structural and non-structural characteristics of the buildings. The interventions are calibrated regarding to the hazard and to the expected performances. The technical code gives the tools useful for the evaluation of the vulnerability and its reduction through interventions.

EXPOSITION

The use of the territory is designed by acting on the building distribution and density, on infrastructures, on the use destinations. Moreover, the protection level is increased by increasing the risk knowledge and improving the behaviors in case of earthquake.

SEISMIC RISK REDUCTION

Vulnerability assessment



PROBABLISTIC APPROACH TO THE ASSESSMENT

1. **PROBABILISTIC MODEL OF THE CAPACITY**
   K samples of the structure nominally identical, but statistically different

2. **PROBABILISTIC MODEL OF THE DEMAND**
   Set of spectra / accelerograms with increasing intensity (e.g. PGA=0.1,0.2,..1.0 g)

3. **DEFINITION OF PERFORMANCE LEVELS (PL)**
   e.g. ductility assumed as Damage Measure (DM)

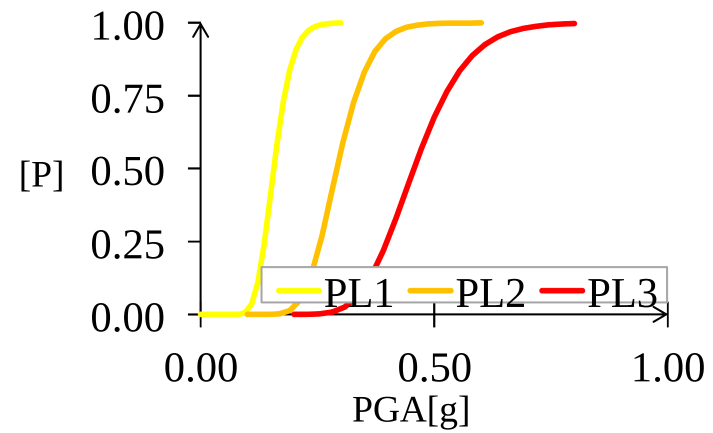
$$D_{pier,1} \rightarrow \mu_\delta = 1$$

$$D_{pier,2} \rightarrow \mu_\delta = 2$$

$$D_{pier,3} \rightarrow \mu_\delta = 4$$

4. **DEVELOPMENT OF FRAGILITY CURVES FOR DIFFERENT PERFORMANCE LEVELS**

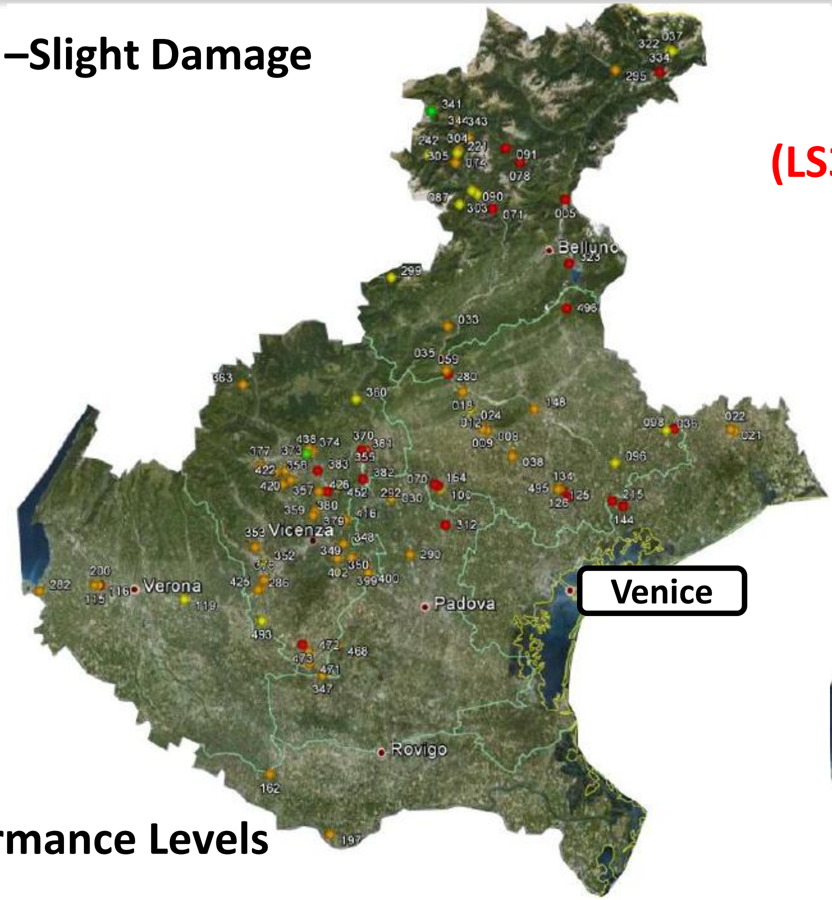
$$P_{f,PL}(a) = \Pr[D(G(\mathbf{p}), S_a \mid a) > d_{PL}] = \int_{D(a)>d_{PL}} f_D(d \mid a)\, \mathrm{d}d$$



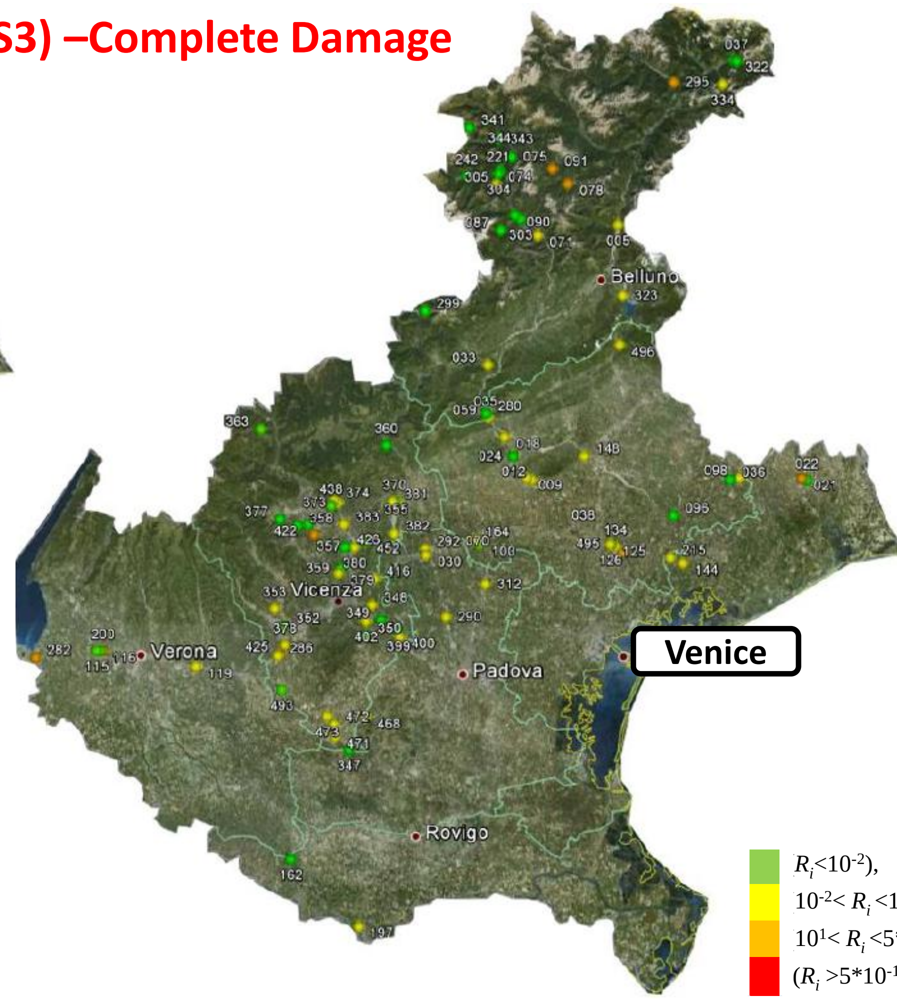
Fragility curve: cumulative probability density function, giving the probability of exceeeding a pre-defined performance level (PL) for different values of seismic intensity measure (IM).

Seismic risk assessment

(LS1) –Slight Damage



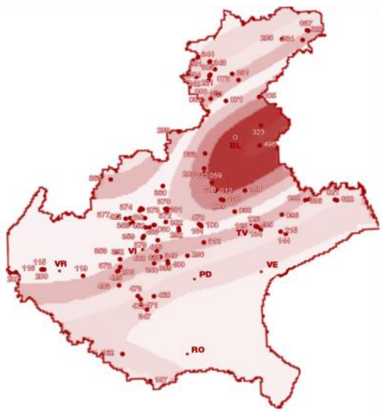
(LS3) –Complete Damage



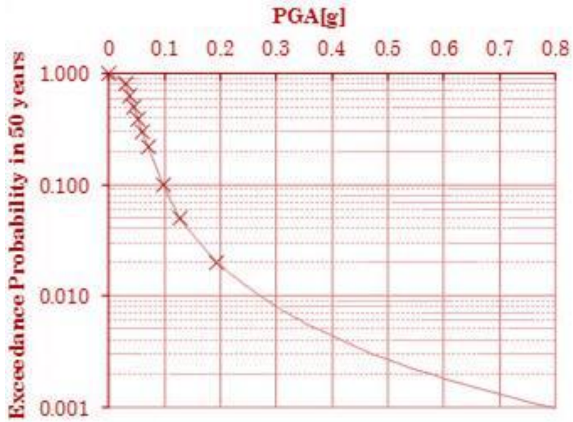
Performance Levels

PIERS	LS1 (Sligth Damage)	LS2 (Severe Damage)	LS3 (Extensive/Complete)
Ductile flexural behaviour	θ_{yn}	$2/3\theta_u$	θ_u
Brittle shear behaviour	-	-	θ_s

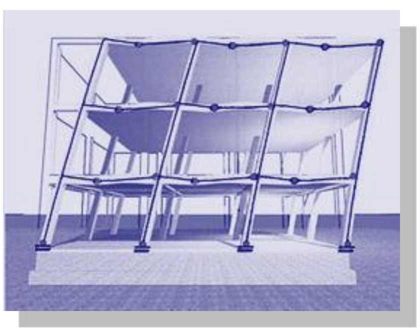
Seismic risk assessment



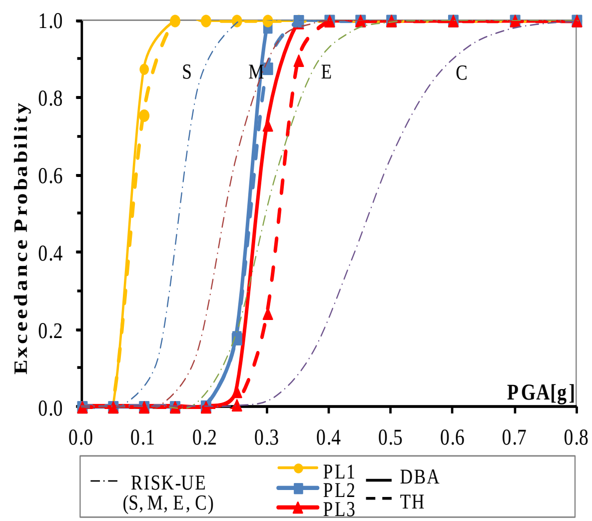
EXPOSURE



HAZARD CURVE



VULNEARABILITY



FRAGILITY CURVE

The risk related to a pre-defined damage level d_{PL} is obtained by convolution of the probability density function of the hazard:

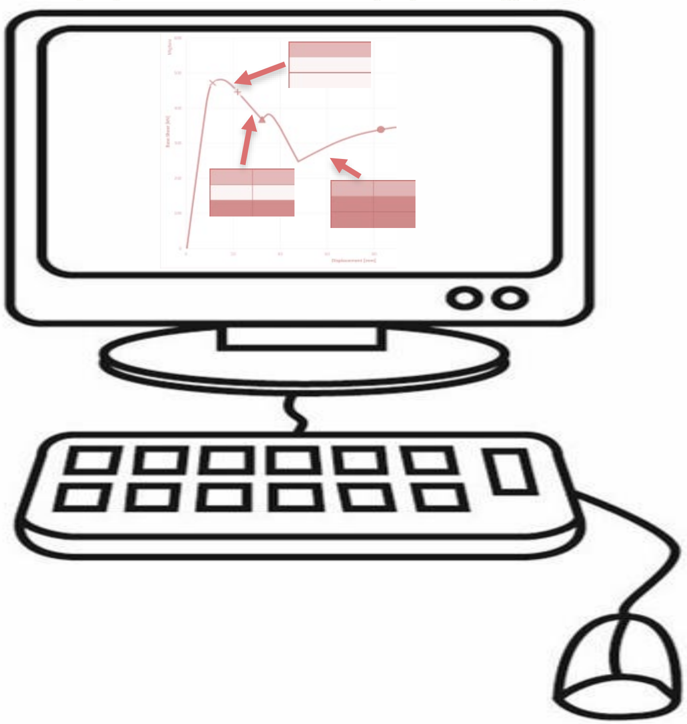
RISK INDEX

	DBA	TH
PL1	0.127	0.108
PL2	0.011	0.010
PL3	0.009	0.007

$$P_{PL} = \int_{IM} P(D > d_{PL} \mid IM) \left| \frac{d\lambda(IM)}{dIM} \right| dIM$$

where $P(D>d_{PL}|IM)$
Is the fragility associated to a damage level d_{PL} and H(IM) is the hazard curve

Thanks for your kind attention!



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