Structural Modeling of Historical Constructions

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Contents

- NIKER
- What is Masonry?
- “Traditional” Approaches for Masonry Analysis
- Difficulties in practice
- Applications
Linear elastic analysis?

- “Ut tensio sic vis” or $\sigma / E = \varepsilon$ is the elasticity law established by R. Hooke in 1676. The theory is so extensively used that its limitations and deficiencies are often forgotten. This is in opposition with early forms of limit analysis.

Cantilever beam according to Galileo (1638) and evolution of the “hypothesis” for the stress distribution at AB.

Retaining wall according to Coulomb (1773).
Or non-linear analysis?

- Structural collapse does not generally coincide with the appearance of the first crack or localized early crushing → elasticity theory is a step back with respect to limit analysis?

- Non-linear analysis includes the full loading process, from absence of loading, through behavior under service loading, until collapse → the most advanced form of structural analysis?

- Interest growing since 1970’s → it remains a field for specialists due to complexity (knowledge) and costs (time) involved

- Possibilities are immense and it is often included in commercial software, but an incorrect use can be very dangerous
Analysis Methods

Analysis

Static
- Linear Static
- Pushover Analysis (Non-Linear)
  - Adaptive Pushover

Dynamic
- Limit Analysis
- Modal Analysis
  - Incremental Dynamic
- Non-linear Dynamic
NIKER Deliverables

- Simplified and complex models of in- and out-of-plane response to be implemented in global analyses

- Development of reliable numerical models and assessment of connections and substructures

- Parametric study of buildings according to geometry, intervention techniques, stiffness of horizontal elements, connections

- Reliably quantification of building performance and response parameters for use in seismic assessment and design
Development of reliable numerical models and assessment of connections and substructures: Connections

Unstrengthened vs. timber-laced

T-Structure masonry wall

Cross vaults
Development of reliable numerical models and assessment of connections and substructures: Wall to wall connections
Development of reliable numerical models and assessment of connections and substructures: Dissipative connections

Validation
Parametric study of buildings: Floor stiffness

M1: unidirectional floor simply supported to the shear walls

Walls are 3 m high, 4 m long and 50 cm thick, and equally spanned at 3.5 m

M2: bidirectional floor (slab) simply supported in a box-like wall structure

M3: unidirectional floor in a box-like wall structure simply supported to the shear walls
## Parametric study of buildings: Material properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Changed value</th>
<th>Mode shape X</th>
<th>Frequency X [Hz]</th>
<th>Mode shape Y</th>
<th>Frequency Y [Hz]</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UNREINFORCED MODEL</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>Profile of planks</td>
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<td>Properties of wooden</td>
<td>C24 E=11GPa G=0.69GPa</td>
<td>8.18</td>
<td>6.10</td>
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<tr>
<td>Properties of masonry</td>
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<tr>
<td><strong>TIMBER LACED MODEL</strong></td>
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<td>Profile of joists</td>
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<td>9.10</td>
<td>6.70</td>
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<td>Profile of planks</td>
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</table>
## Parametric study of buildings: Material properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Lower value</th>
<th>Reference value</th>
<th>Upper value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus of the walls</td>
<td>0.5(E_{\text{walls,ref}})</td>
<td>(E_{\text{walls,ref}} = 1.00) GPa</td>
<td>2.0(E_{\text{walls,ref}})</td>
</tr>
<tr>
<td>Young’s modulus of the floors</td>
<td>0.1(E_{\text{floors,ref}})</td>
<td>(E_{\text{floors,ref}} = 0.16) GPa</td>
<td>10(E_{\text{floors,ref}})</td>
</tr>
<tr>
<td>Compressive strength</td>
<td>0.5(f_{c,\text{ref}})</td>
<td>(f_{c,\text{ref}} = 1.00) MPa</td>
<td>2.0(f_{c,\text{ref}})</td>
</tr>
<tr>
<td>Compressive fracture energy</td>
<td>0.5(G_{c,\text{ref}})</td>
<td>(G_{c,\text{ref}} = 1.00) N/mm</td>
<td>2.0(G_{c,\text{ref}})</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>0.5(f_{t,\text{ref}})</td>
<td>(f_{t,\text{ref}} = 0.10) MPa</td>
<td>2.0(f_{t,\text{ref}})</td>
</tr>
<tr>
<td>Tensile fracture energy</td>
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<td>(G_{t,\text{ref}} = 0.05) N/mm</td>
<td>2.0(G_{t,\text{ref}})</td>
</tr>
</tbody>
</table>

**Load pattern**: Displacement proportional to the first mode
Parametric study of buildings: Connections

Reference model

X direction

Y direction

UNIVERSITY OF MINHO
Civil Engineering Department

Paulo Lourenço
Parametric study of buildings: Connections

**Weakened Connections**

*X direction*

*Y direction*
Reliable seismic assessment: Displacement based design

Real Structure

2 DoF  4 DoF

Displacement Shape

middle-storey DoFs showed displacements that can be consider linear with those at storey levels:

- Schematization with 2 DoF is enough accurate
- Displacement shape can be identified by normalized displacement at first floor ($\Delta_{FF}/\Delta_{top}$)

Results of dynamic analyses:
- Both the directions
- For the several levels of PGA

$\Delta_{FF}/\Delta_{top} \approx 0.65 \div 0.70$

- Effective Height $\approx 0.7 \, H_{tot}$
- Effective Mass $\approx 0.8 \, M_{tot}$
WHAT IS MASONRY?
What is masonry?
What is masonry?

Masonry can be defined as a material with visible internal structure.
Why is this relevant for mechanics?

Shear testing of stone joints

Stress-strain relation

Failure surface

Dilatancy

\[
\tau = -0.179 \sigma \\
\tau = -0.618 \sigma \\
\tau = -0.561 \sigma
\]

\[
r^2 = 0.866 \\
r^2 = 0.975 \\
r^2 = 0.985
\]

Normal stress (N/mm²) vs. Shear stress (N/mm²)
Why is this relevant for mechanics?

Stone walls

Collapse Mechanism and Strength
- Regular – $\tan \phi = 0.4$
- Irregular – $\tan \phi = 0.3$
- Rubble – $\tan \phi = 0.2$
“TRADITIONAL” APPROACHES FOR MASONRY ANALYSIS
Modeling masonry – Material Level

Sophisticated models require advanced material characterization. Much experience had been gained in the last decade.
Modeling Approaches – Structural Level (I)

Beam elements model  
3-degree of freedom model  
Macro-block model

STRUCTURAL COMPONENT MODELS
Modeling Approaches – Structural Level (II)

Shear wall

Wall with out of plane behavior

Church Settlements

STRUCTURAL MACRO-MODELING / FINITE ELEMENT METHOD
Modeling Approaches – Structural Level (III)

Shear wall (in plane behavior)

Wall with out of plane behavior

STRUCTURAL MICRO-MODELING / FEM, DEM, LIMIT ANALYSIS
HOMOGENIZATION TECHNIQUES
HOMOGENISATION
(or from micro- to macro-modeling)

\[ \tau_3 > \tau_2 > \tau_1 > \tau_0 = 0 \]

Hill type criterion

Rankine type criterion

Masonry Anisotropy

Biaxial Testing
HOMOGENISATION (or from micro- to macro-modeling)

The failure surface of the structure is normally not known, but only calculated for each stress path.
MULTI-PATH SOLUTION AND COMPARISION OF HOMOGENISED COLUTION WITH EXPERIMENTAL RESULTS (PAGE 1981)

1) Failure surface units: Rankine ($f_t=0.1\ N/mm^2\ f_c/f_t=20$)

2) Failure surface mortar: Mohr Coulomb ($f_t=0.15\ N/mm^2\ f_c/f_t=15$)

$e_v=1.5$

$e_h=1.5$

$b=3.3a$
LIMIT ANALYSIS HOMOGENIZATION APPROACH

Lower Bound

Upper Bound

Results

0 2.5 5 7.5 10 12.5 15
Horizontal Displacement [mm]

50 100 150 200 250 300
Horizontal Load [kN]

Experimental
Limit Analysis
EXTENSION TO SLABS

Lower Bound

Upper Bound

Results

University of Plymouth data
Lourenço model (1997)
Proposed homogenized model

max displacement [mm]

Panel SB01
Panel SB02
Panel SB03
Panel SB04

Simply supported edge
Clamped edge
Free edge

p [kN/m²]

0.0
0.5
1.0
1.5
2.0
2.5

2.66 U.B.
2.25 L.B.
APPLICATION TO A SCHOOL BUILDING IN ITALY

Limit analysis FEM

FEM standard model

Results

Elastic-plastic analysis
Limit analysis

shear at the base [kN]

0 500 1000 1500 2000 2500 3000 3500 4000 4500 5000

0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
displacement [mm]
Sensitivity Analysis (I)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$k$</th>
<th>$f_i$</th>
<th>$c$</th>
<th>$\tan \phi$</th>
<th>$f_m$</th>
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<tr>
<td>Divided by 1.25</td>
<td>52.84</td>
<td>55.92</td>
<td>55.70</td>
<td>50.50</td>
<td>50.79</td>
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<tr>
<td>Multiplied by 1.25</td>
<td>58.80</td>
<td>55.47</td>
<td>55.08</td>
<td>60.46</td>
<td>63.17</td>
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<tr>
<td>Difference to original</td>
<td>6.2%</td>
<td>1.1%</td>
<td>0.6%</td>
<td>9.6%</td>
<td>14.1%</td>
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</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$G^I$</th>
<th>$G^{II}$</th>
<th>$G^C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Divided by 2.0</td>
<td>55.88</td>
<td>56.16</td>
<td>46.68</td>
</tr>
<tr>
<td>Multiplied by 2.0</td>
<td>55.19</td>
<td>55.54</td>
<td>65.54</td>
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<tr>
<td>Difference to original</td>
<td>0.9%</td>
<td>1.4%</td>
<td>18.6%</td>
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</table>
## Sensitivity Analysis (II)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$E$</th>
<th>$f_{tx}$</th>
<th>$f_{ty}$</th>
<th>$f_{mx}$</th>
<th>$f_{my}$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
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<tbody>
<tr>
<td>Divided by 1.25</td>
<td>162.4</td>
<td>167.8</td>
<td>168.0</td>
<td>162.8</td>
<td>146.0</td>
<td>180.5</td>
<td>164.6</td>
</tr>
<tr>
<td>Multiplied by 1.25</td>
<td>172.9</td>
<td>168.3</td>
<td>168.0</td>
<td>172.7</td>
<td>189.4</td>
<td>148.5</td>
<td>172.7</td>
</tr>
<tr>
<td>Difference to original</td>
<td>3.4%</td>
<td>0.2%</td>
<td>0.1%</td>
<td>3.2%</td>
<td>13.1%</td>
<td>11.7%</td>
<td>2.7%</td>
</tr>
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\gamma$</th>
<th>$G_{fx}$</th>
<th>$G_{fy}$</th>
<th>$G_{fex}$</th>
<th>$G_{fey}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Divided by 2.0</td>
<td>180.3</td>
<td>167.5</td>
<td>167.8</td>
<td>162.5</td>
<td>151.1</td>
</tr>
<tr>
<td>Multiplied by 2.0</td>
<td>151.8</td>
<td>169.3</td>
<td>168.4</td>
<td>169.5</td>
<td>179.0</td>
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<td>Difference to original</td>
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<td>0.7%</td>
<td>0.2%</td>
<td>3.3%</td>
<td>10.1%</td>
</tr>
</tbody>
</table>
Sensitivity Analysis (III)

1. Moderate influence of material data
2. Low influence of vertical earthquake action
3. Reduced scale models (with Cauchy law of similitude) is conservative
DIFFERENT MODELS = DIFFERENT RESULTS ??
Static Analysis Methods (I)

- **Linear Elastic Analysis**
  elastic properties + maximum admissible stress

- **Kinematic Collapse Mechanism Analysis**
  inelastic properties = friction angle + tensile and compressive strengths

- **Static Thrust Line Analysis**

- **Non-linear Analysis (Physical and Combined)**
  FULL inelastic properties (ft = 0 and ft ≈ 0) + elastic properties
Max. 0.64 N/mm²
Linear Elastic

Min. -1.0 N/mm²
Linear Elastic

Min. -5.4 N/mm²
Phys. Non-Linear

Min. -5.4 N/mm²
Comb. Non-Linear

Kin. load factor : 1.8
Failure Mechanism

Geo. load factor : 1.2
Thrust Line

Static Analysis Methods (II)
Static Analysis Methods (III)

Vertical displacement at quarter span (mm)

Load factor

- Limit analysis
- $ft = 0$, Physically non-linear
- $ft = 0$, Physically / Geometrically non-linear
- $ft = 0.2 \text{ N/mm}^2$, Physically non-linear
- $ft = 0.2 \text{ N/mm}^2$, Physically / Geometrically non-linear
More on Static Analysis Methods…

Safety factor: 124%

115%

103%

77%
Using the Right Analysis Method!

- **Dead load:** 6.0 kN/m²
- **Live load:** 1.5 kN/m²

### Nonlinear Analysis
- **1 storey**
- **2 storeys**
- **3 storeys**

### Elastic Analysis (q=1.5)
- **1 storey**
- **2 storeys**
- **3 storeys**

- **Cannot be used**
- **Can be used for hard soils**
- **Can be used for good soils**
KEEP IT SIMPLE!
Modeling of Historical Structures (I)

- In contrast to modern structures, it is not straightforward to define the conditions under which a given idealization of the geometry is applicable.
- Usually, the geometry of historic masonry structures is rather complex as there is no distinction between decorative and structural elements. Therefore, as a first impression, it would seem reasonable to advise the use of three-dimensional elements. But this impression is erroneous!
- The practical analysis of historic masonry structures entails severe simplifications. As a rule, the geometric idealization should be kept as simple as possible, as long as it can be considered adequate for the problem being analyzed. In particular:
  - Fully three-dimensional models are usually very time-consuming with respect to preparation of the model, to perform the actual calculations and to analyze the results. Several authors use eight-node bricks, with one element over the thickness of the walls or vaults. The errors associated with such a discretization are very large, even for linear elastic analysis, yielding meaningless results;
  - The results of models incorporating shell elements are fairly difficult to analyze owing to the variation of stresses along the thickness of the elements. In addition, the large thickness of the structural elements may yield a poor approximation of the actual state of stress;
  - Increasing the detail and size of the model might result in a large amount of information that blurs the important aspects.
Nice pictures (several man-months of work)…
Modeling of Historical Structures

- If possible, it seems preferable: (a) to use two-dimensional models rather than three-dimensional models; (b) to avoid using shell elements in areas important for the global behavior of the structure; and (c) to model structural parts and details instead of modeling complete and large structures.
Modeling of Historical Structures

- Details…

Cloister in Outeiro. Three different simple models to understand the damage: 2D, Shell and 3D
Lethes Theatre, Faro
Holy Christ Church, Outeiro
São Torcato Sanctuary, São Torcato
Saint Francis Church, Horta, Azores
Donim Bridge, Donim
Typical downtown construction of Lisbon
CONCLUSIONS
CONCLUSIONS

- Structural analysis (quantitative output) is not the only information considered for decision of safety, even if it is very relevant. Subjectivity and uncertainty are there. Use also other sources of knowledge.

- Advanced tools for masonry structural analysis are available. Information on material data to feed advanced models is available.

- Difficult task, which requires training and specific skills: (a) scatter of properties, lack of original design elements / Non-conforming execution, deficient structural connections, load transfer, …; (b) different materials, technologies, …; (c) lack of education and non-applicable codes.

- Different approaches can provide different results. Be sure to validate the models and use a correct approach.

- Keep it simple, if you can 😊!
Structural Modeling of Historical Constructions

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